DIVERSIFICATION DASHBOARD

March 2018

Diversification Ratios®

TOBAM's Diversification Ratio® (DR) measures to what extent a portfolio is diversified. The DR^2 (square of the diversification ratio) measures the number of independent sources of risk to which a portfolio is exposed.
As the table shows, the "broad market" indices do not fully utilise diversification capabilities. In addition to a snapshot of each market's DR^2, the table shows the DR^2 of a well-diversified portfolio, and the fraction of available diversification used by the index.

Universes	DR^2 Index diversification	DR^2 Maximum Diversification®	% diversification used by index
MSCI All Countries World	4.41	13.63	32.4%
MSCI Canada	5.80	12.79	45.4%
MSCI World	4.20	11.44	36.7%
MSCI US Equity	3.80	10.36	36.7%
MSCI Emerging Markets	3.90	8.90	43.9%
MSCI Pacific Ex-Japan	3.48	8.19	42.5%
MSCI EMU	2.84	6.36	44.7%
MSCI UK Equity	3.88	6.24	62.2%
MSCI Japan	2.62	6.05	43.3%
BofA Merrill Lynch US Corporate & High Yield	4.49	6.61	67.9%
BofA Merrill Lynch Global High Yield	5.96	7.25	82.2%

Source: TOBAM, figures as of February 28, 2018.

Characterizing portfolios through their correlations

We illustrate in this dashboard some concepts and results from our latest working paper, 'Portfolio rho-presentativity' [1]. This is the second in a series, dedicated to this working paper.

In this month's Diversification Dashboard, we introduce a method of calculating the Diversification Ratio® (DR®) that does not require portfolio weights. We define the concept of a portfolio's correlation spectrum and then present a methodology to compute the realized DR® of any portfolio based only on its historical returns. We use empirical results from the US equity universe to illustrate these two concepts.

Introduction

While "past performance is not indicative of future returns", the record of a fund's net asset values is the fund data which is the most easily accessible to the public. Sharpe's (1988) return-based style analysis uses a regression of the fund's historical returns over a set of specific factors, so that the coefficients of the regression estimate past exposures. This approach to a fund's performance analysis has proved very popular and is now offered as a standard in many if not most financial providers' software.

Recently, [1] proposed an alternative definition of exposure, based on correlations instead of weights, that also uniquely defines a portfolio. Focusing on correlation yields fresh insights into the characteristics of a portfolio. In particular [1] demonstrates that the correlations of a portfolio with the assets of its investment universe are informative as to its level of diversification: there is indeed a theoretically

constructed portfolio whose returns can be used as a benchmark to estimate any other portfolio's Diversification Ratio®.

In the following section, we present the results of our 2017 publication that are relevant to return-based analysis. The concepts of the correlation spectrum and the long-short Maximum Diversification® portfolio are explained using numerical examples. We provide an empirical analysis of the US equity funds universe to illustrate the practical applications of our research.

1. The correlation spectrum

The Diversification Ratio® of a portfolio is defined in Choueifaty et al. (2008) as:

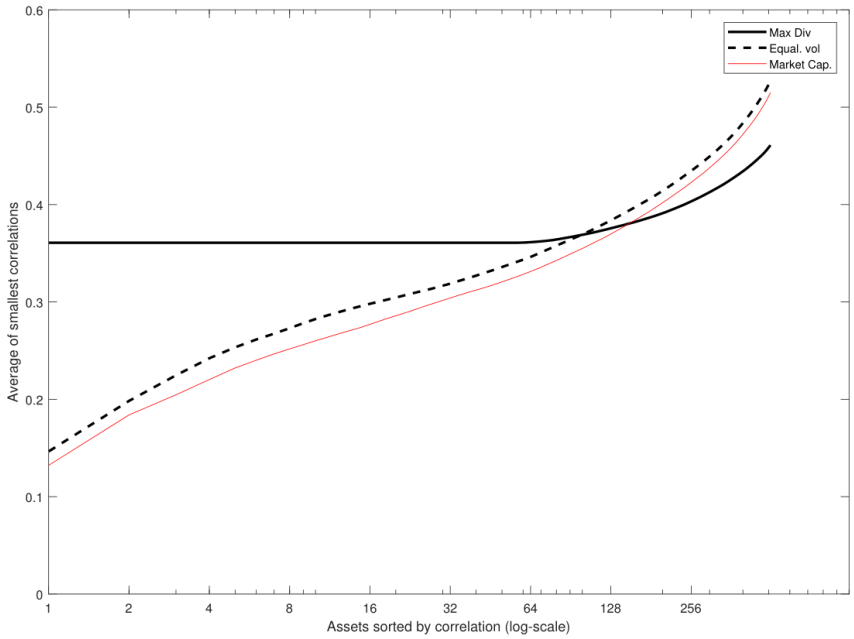
$$DR(w) = \frac{\sum_i w_i \sigma_i}{\sigma(w)},$$

where σ_i is the volatility of asset i and $\sigma(w)$the volatility of the portfolio with weights vector w.

To compute the Diversification Ratio® from this definition, weights are needed. The latest results in our 2017 article circumvent this limitation and provide the foundations for an alternative method, without the need for weights.

We consider the correlation spectrum $\rho(w)$, which is the vector of correlations of a portfolio of weights w with every asset in its investment universe. Differing portfolios (up to leverage) will yield different correlation spectra. A way to visualize the spectrum is to sort it before plotting it, with the assets on the x–axis and the correlations or their average (which is more robust) on the y-axis. In order to facilitate comparisons between different portfolios - the exact order of the assets is unimportant when measuring the level of diversification, however the shape of the spectrum is. The examples below are computed on the US equity market, with daily data from 2013-2017.

Figure 1: Correlation spectra



Source: Bloomberg, computations by TOBAM

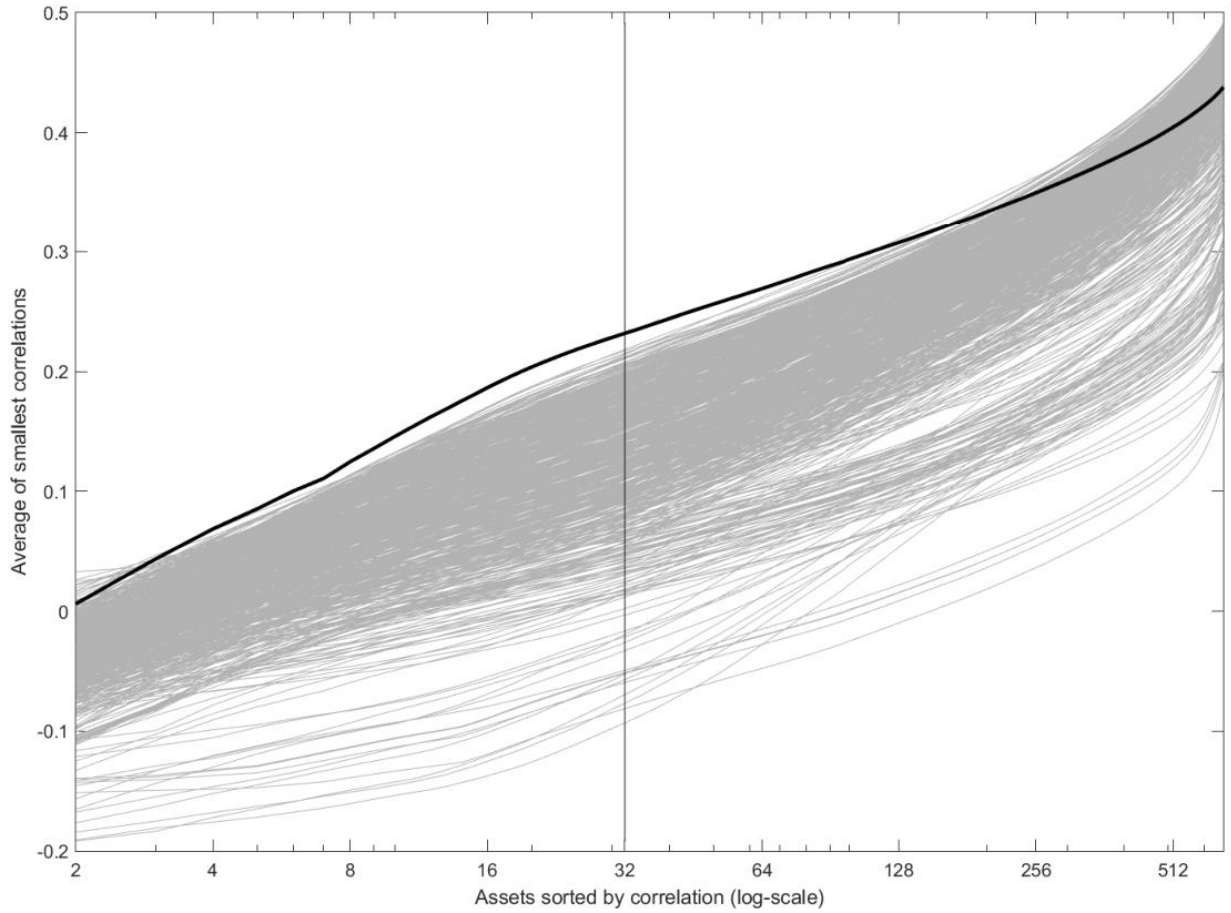
As described in [1], well-known risk-based portfolios have remarkable spectra, notably:
- the equal volatility weighted allocation maximises the average correlation to all assets,
- the Maximum Diversification® portfolio maximises the minimal correlation(s).

Figure 1 illustrates the typical shape of these ex-ante portfolios on real data. For comparative purposes, the market capitalization portfolio is represented as well.

While, as in figure 1, the spectrum can be plotted ex-ante for given portfolio weights, it can also be computed ex-post, using only the realizations of the portfolio's returns. As such, it provides a way to assess realized diversification, and to compare portfolios without knowing their exact composition.

Figure 2 below illustrates the realized correlation spectra of US equity funds with the assets included in the MSCI USA index. The spectrum of our Anti-Benchmark® US strategy is illustrated using the bold line.

Figure 2: Correlation spectra of US equity funds NAVs with each asset included in the MSCI USA index



Source: TOBAM. Correlation of US Equity funds (AuM above $300mn: 1700 funds over the period from Jan 2013 to Nov 2017), to assets included in MSCI USA Index: 700 assets, with enough observation on the period. Weekly data were used in order to avoid delayed observations and off-hours trading. The x-scale is logarithmic. Of note: given the implementation constraints on the Maximum Diversification® optimization program, we are in practice, maximizing the average of the 32 smallest correlations (as shown in the vertical line in figure 2). See [1] for more details. For illustrative purpose only.

As could be expected from its construction methodology, TOBAM's Anti-Benchmark® US strategy (shown in bold) displays the highest minimal average correlation. We can see that its average correlation to the stocks it is least correlated with, is above that of its peers at almost any point.

2. Computing realized Diversification Ratios® from historical returns

As a consequence of the choice of using correlations as the method of measuring exposure, [1] demonstrates that the realized Diversification Ratio® of any portfolio can be estimated from the correlation of the time series of its returns with those of the long-short portfolio that maximises the Diversification Ratio®.

$$DR(w) = DR(\overline{w})\rho(\overline{w}, w),$$

with $\overline{w} = \frac{\Sigma^{-1}\sigma}{\|\Sigma^{-1}\sigma\|}$, Σ the covariance matrix of the stocks and σ their volatilities.

This long-short reference portfolio[1] has, by definition, an equal correlation to every asset in its universe. From [1]'s remarkable result, the time series of the returns of this theoretical portfolio is now all we need to compute the Diversification Ratio® of any other portfolio with assets from the same universe observed on the same time-frame.

The table below gives the results of the estimation using full in-sample information for our universe of funds with a focus on AB US®.

	Best possible long-only reference Maximum Diversification® portfolio	Live Anti-Benchmark® US (best actual DR)	Top 5% of the funds (most highly correlated)	Median	Bottom 5% of the funds (most lowly correlated)
Correlation with reference portfolio	0.45	0.34	0.25	0.21	0.12
Realized Diversification Ratio®	3.7	2.7	2	1.7	1

Source: TOBAM.

The first column illustrates the maximum theoretical measures for the long-only portfolio that would maximize the Diversification Ratio® using the best estimate of the covariance matrix, which can only be measured retrospectively. These are the same inputs as the long-short reference portfolio, but the long-only constraint means the L/S reference portfolio is more diversified.

No long only fund can exceed the values of the first column by definition, and in practice, only few approach it: **90% of the funds have a correlation between 0.12 and 0.25 i.e. a Diversification Ratio® between 1 and 2**[2] .

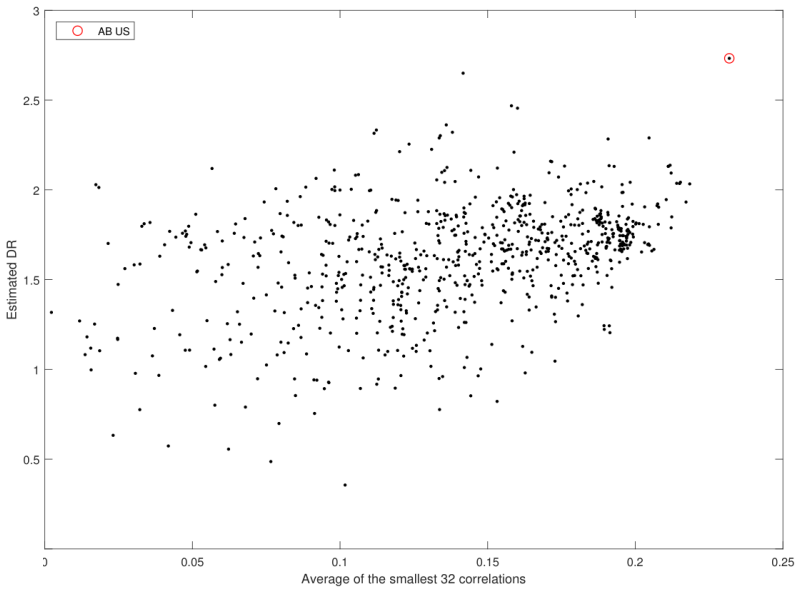
Among the universe of live US equity funds, the highest Diversification Ratio® level observed is seen in Anti-Benchmark® US (2.7 over the period). Calculations from our internal database, using real weights of the live AB® US strategy produces a DR® of 2.76, the returns-based estimate of 2.7 in the table above seems accurate.

[1]: *Of note this L/S reference portfolio can only be defined ex-post, from the historical data series. It cannot be built ex-ante.*

[2] *The distribution of the correlations of the funds to the long-short reference portfolio, are overall, low: the maximum is about 0.3 for the Anti-Benchmark® US strategy as expected. Diversification Ratios® never go below 1 for long-only portfolios, so the presence of Diversification Ratios® below 1 indicates either estimation bias, or portfolios which are either long-short or invested outside this universe (these funds number less than 5% of the total).*

Conclusion: using past returns to assess diversification

Figure 3: Combining correlation spectrum and realized DR® to assess diversification



Well-diversified funds

Poorly-diversified funds

Source: TOBAM & Bloomberg. For illustrative purpose only.

Figure 3 combines the two previous results to present an integrated assessment of diversification, from the observation of past returns alone. Both the correlation spectrum and the estimated DR® demonstrates the unique nature of the Anti-Benchmark® US. As it is the only strategy with the explicit utility function to maximize the Diversification Ratio®, AB® US is clearly distinct from its peers in Figure 3. The same methodology can be used to classify the universe also, with well diversified funds occupying the upper right corner of the plane and poorly diversified funds occupying the lower left.

To sum up, we have shown that characterizing portfolios, through their correlations with their investment universe, yields interesting insights into their characteristics. The correlation spectrum and the realized diversification are computable from the public record of price returns, and they allow us to diagnose individual portfolios – something that, up to now, was not possible without using proprietary information.

Portfolios are usually characterized by a wide set of indicators: performance returns, volatility, Sharpe ratio, max drawdown, or relative indicators such as alpha, tracking error, information ratio, beta, Sortino Index, DownSideRisk…
Diversification is a well-known concept, which is often promoted and highlighted by investment professionals when describing their investment process or strategy. The importance of correlation to determine risk exposure is recognized by most professionals of the asset management industry, but the use of a proper measure to qualify and quantify the quality of the diversification of a portfolio is yet to become mainstream.
We believe that the approach proposed by "*Portfolio Rho-Presentativity*" could facilitate a more widespread adoption of the Diversification Ratio® as a core indicator of a portfolio's characteristics.

Reference:

[1] Froidure T., K. Jalalzai and Y. Choueifaty, (2017), "Portfolio rho-presentativity", working paper, SSRN.

For more information

TOBAM is an asset management company offering innovative investment capabilities designed to increase diversification. Its mission is to provide rational and professional solutions to long term investors in the context of efficient markets.

The Maximum Diversification® approach, TOBAM's flagship investment process founded in 2006, is supported by original, patented research and a mathematical definition of diversification and provides clients with diversified core exposure, in both the equity and fixed income markets.

In line with its mission statement and commitment to diversification, TOBAM also launched a separate activity on cryptocurrencies in 2017.
Diversification is our only bet.

TOBAM currently manages US$9.5 billion (at December 29, 2017). TOBAM's team is composed of 49 professionals.

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