TOBAM
CORE INVESTMENTS

DIVERSIFICATION DASHBOARD

JULY 2018

Diversification Ratios®

TOBAM's Diversification Ratio® (DR) measures to what extent a portfolio is diversified. The DR^2 (square of the diversification ratio) measures the number of independent sources of risk to which a portfolio is exposed.
As the table shows, the "broad market" indices do not fully utilise diversification capabilities. In addition to a snapshot of each market's DR^2, the table shows the DR^2 of a well-diversified portfolio, and the fraction of available diversification used by the index.

Universes	DR^2 Index diversification	DR^2 Maximum Diversification®	% diversification used by index
MSCI All Countries World	4.52	14.80	30.6 %
MSCI World	4.26	13.18	32.3 %
MSCI US Equity	3.55	10.55	33.7 %
MSCI Emerging Markets	4.27	9.34	45.7 %
MSCI Pacific Ex-Japan	3.69	8.34	44.3 %
MSCI EMU	3.14	7.43	42.3 %
MSCI Japan	3.09	7.26	42.5 %
MSCI UK Equity	3.74	6.51	57.4 %
BofA Merrill Lynch US Corporate & High Yield	4.50	6.79	66.3 %
BofA Merrill Lynch Global High Yield	6.83	8.86	77.1 %

Source: TOBAM, figures as of June 28, 2018.

Maximally ρ-presentative Portfolios

In this dashboard, we introduce some concepts and results from our recently released academic paper, 'Portfolio Rho-presentativity' [1]. This dashboard is the third in a series dedicated to this paper.

Abstract

Investors often look for benchmarks that best represent an entire asset class and the capitalization weighted index is usually regarded as such a "representative" benchmark.

In this dashboard, we therefore introduce the concept of maximally ρ-presentative portfolio that allows an investor to hold a portfolio that is

- maximally exposed to all assets (exposures being measured by correlations instead of weights),
- and fairly diversified as measured by the Diversification Ratio (DR).

Furthermore, using this concept we also prove that investors willing to maximize the exposure of their portfolio to all assets should only consider long-only portfolios.

As we shall see, maximally ρ-presentative portfolios are quite rare but, in theory, encompass several well-known alternative beta investment strategies.

In practice, very few portfolio implementations are actually maximally ρ-presentative. In particular, the TOBAM Anti-Benchmark® US equity strategy stands out amongst thousands of comparable US equity funds, as the *only* portfolio that could claim it has been maximally exposed to the MSCI USA universe, while also offering the highest level of realized diversification, thus fulfilling its mission.

1. Introduction and Outline

The usual weight representation of a portfolio may not always indicate to what degree the portfolio is exposed to a particular asset. For instance, not holding any financial stock does not necessarily mean no exposure to the financial sector. This remark prompts in a natural way the concept of the *correlation spectrum* that measures exposures through correlations. Let us recall this key concept that was introduced in [1, Definition 2.1] but also in a previously published dashboard: given an investment universe, the correlation spectrum of a portfolio with weights w is the vector $\rho(w)$ of correlations of all assets to that portfolio. This vector offers an alternative but equivalent representation to the weights w and is not limited to long-only portfolios.

Now, let us introduce the notion of *maximally ρ-presentative portfolio*, which is a portfolio that maximizes an aggregate exposure to all assets as measured by a (real-valued) function f of the spectrum, namely $f(\rho(w))$. We shall get back to this definition in Section 2 where we shall also formalize the following important properties of *maximally ρ-presentative portfolios*:

- they are necessarily *long-only,*
- they are *quite rare,*
- they satisfy an original *geometric property* and
- they are *well diversified*.

To establish most of these results, the key is to find an explicit characterization of these maximally ρ-presentative portfolios that is independent of f: these are *essentially* the long-only portfolios whose exposures form a non-increasing function of their weights.

However, as we shall see in Section 3, even though these portfolios are rare, *this new class encompasses many well-known alternative beta portfolios*, including the Most Diversified Portfolio (MDP), the Equal Volatility Weighted (EVW, aka Risk Weighted) and the Equal Risk Contribution (ERC, Risk Parity). To observe this, we provide alternative definitions of all these portfolios in terms of $f(\rho(w))$. Similar results are established in [1] for other well-known alternative beta strategies such as the Minimum Variance (MV) and Equal Weight (EW) portfolios.

We end this dashboard in Section 4 by presenting some empirical results from the US equity universe that show that, between 2013 and 2017, *the TOBAM Anti-Benchmark® US equity strategy has offered the highest level of realized diversification, and has also been the sole candidate for being maximally ρ-presentative* amongst 2278 of its peers, that represent 80% of the net assets invested in US equities.

2. Definition and Properties of Maximally ρ-presentative Portfolios

2.1 Definition

As recalled in the introductory section, the correlation spectrum ***$\rho(w)$*** *is the vector* of correlations of a portfolio of weights w with every asset of the investment universe. Now, given a long-short portfolio w and associated spectrum $\rho(w)$, we consider *any* function f that aggregates all these exposures to get *a single number* ***$f(\rho(w))$*** which measures how *representative* of the universe portfolio w is as a whole.

Definition: *A maximally ρ-presentative portfolio maximizes - amongst all long-short portfolios w - its exposures to all assets through the aggregate view offered by* ***$f(\rho(w))$****.*

We need this function f to be at least:

- increasing to advantage a portfolio that is more exposed to each asset than another.
- symmetric as there is *a priori* no reason for it to change if we permute the exposures/assets.

For simplicity, in this note we skip a third assumption (cf. [1, Definition 4.1] for the details) that also guarantees the existence and uniqueness of such portfolios.

Example: One could view each f generating a maximally ρ-presentative portfolio as providing a particular trade-off between the average and the dispersion of the spectrum of a portfolio. To illustrate this idea, denoting respectively E(v) and Var(v) the mean and variance of a vector v, let us note that

$$f(\rho(w)) = \mathbb{E}(\rho(w)) - \frac{\lambda}{2}\mathbb{V}ar(\rho(w))$$

satisfies our assumptions on f for $\lambda<1$. This draws an obvious parallel between the mean-variance utility criterion used for portfolio construction and the objective maximized to obtain a maximally ρ-presentative portfolio.

2.2 Theoretical Properties

In this section, we state some of the most important properties satisfied by these portfolios and refer to [1] for further details.

Property 1: *A maximally ρ-presentative portfolio is necessarily long-only.*

Idea of the proof: The key is to show that for any portfolio that is not long-only there always exists a long-only portfolio that dominates it in the sense that it is more correlated to each asset.

As the spectrum is well-defined for long-short portfolios, all such portfolios may qualify for being maximally ρ-presentative. However, it is quite remarkable that in the end, portfolios that are maximally exposed to all assets are necessarily long-only. This, in a way, debunks the myth of long-only portfolios as in our approach we do not need to constrain our portfolios to have non-negative weights - as is usually done. The long-only characteristic emerges in the analysis in a natural way.

We now turn to a characterization of the set of maximally ρ-presentative portfolios that is independent on the choice of the aggregation function f:

Property 2: *Maximally ρ-presentative portfolios are exactly those long-only portfolios whose correlations to the assets form a non-increasing sequence of their risk weighted weights.*

Put otherwise, a maximally ρ-presentative portfolio overweighs an asset if and only if it is lowly exposed to it. This property is really binding and justifies why maximally ρ-presentative portfolios are quite rare, which brings us to our next property:

Property 3: *Given a universe of N assets, if one drew, uniformly, the volatility adjusted weights of a long-only portfolio, there is less than 1 over factorial(N) chance to have drawn those of a maximally ρ-presentative portfolio.*

As factorial(N)=1 x 2 x 3 x … x N is a massive number, the probability of finding a maximally ρ-presentative portfolio is miniscule even for small values of the number of assets. For instance, picking a universe of 15 assets, one has less than 1/(1000 billion) chance of finding such a portfolio. If now we consider the S&P500 as our universe of choice, the probability of coming across a maximally ρ-

presentative portfolio is less than 1/(a number with 1135 digits). For comparison, the number of atoms in the universe is estimated to have roughly 80 digits.

We further illustrate this property by depicting in Figure 1 the set of maximally ρ-presentative portfolios of *three assets* within the set of long-only portfolios.

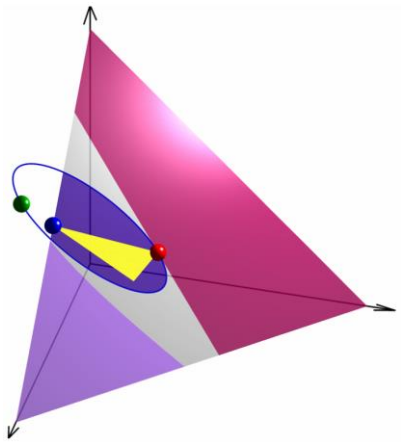

Figure 1: The yellow triangle represents the set of maximally ρ-presentative portfolios of three assets within the set of long-only portfolios that corresponds to the largest triangle. Each axis indicates the weight of each asset. In theory, by Property 3, the surface of the yellow set is at most 1/(1 x 2 x 3) that of the long-only portfolios. The blue and red bullets represent the MDP and the EVW respectively. For further details on other elements of this figure see [1, Figure 3].

Let us now turn to an original geometric property implied by the binding nature of the characterization in Property 2:

Property 4: *The set of maximally ρ-presentative portfolios is a finite union of polytopes (finite union of convex hulls of a finite number of points*[1]*).* We refer to Figure 2 for an illustration.

This property is interesting in practice as each one of these polytopes determines a particular region where all maximally ρ-presentative portfolios are combinations of some special portfolios that are located at its corners. As we can see in Figure 2, The MDP and the EVW are such special portfolios and therefore play a key role in constructing maximally ρ-presentative portfolios.

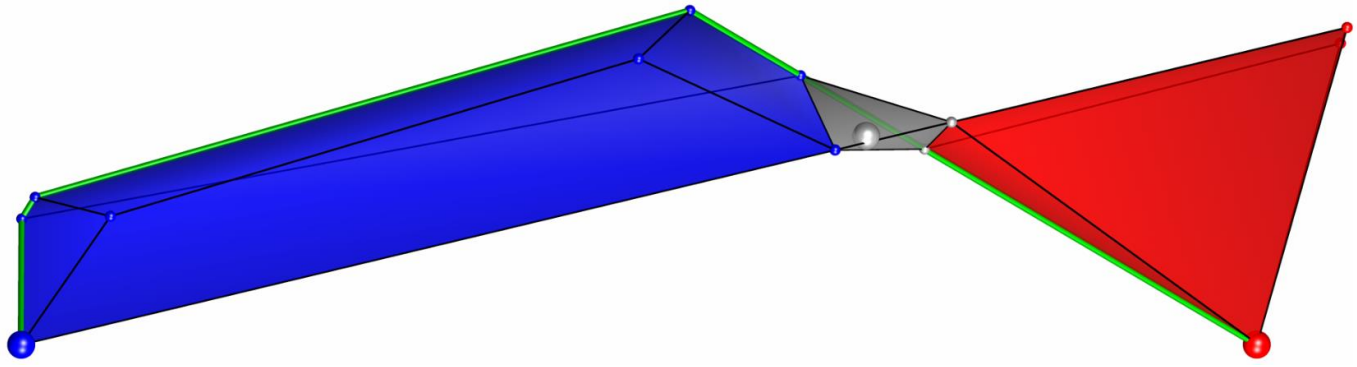

Figure 2: As in Figure 1, omitting the axes, we represent in *dimension three* the set of maximally ρ-presentative portfolios of *four assets*. From left to right the large bullets depict the MDP, ERC and EVW that are indeed maximally ρ-presentative as we are going to see in the next section. We represented, in different colors, the three polytopes that compose the set of maximally ρ-presentative portfolios.

Finally, a maximally ρ-presentative portfolio being more invested in assets it is lowly exposed to, it is rather diversified. In particular, the characterization in Property 2 allows us to prove that maximally ρ-

[1] The convex hull of a set of points is the smallest region determined by an elastic band/surface that encloses that set. A set is convex if it is equal to its convex hull. Thus squares, triangles or cubes are polytopes as they are convex sets that are determined by finitely many corners. On the other hand discs or balls, even though they are convex, are not polytopes as they have infinitely many corners. Finally, a torus is not a polytope as it contains a hole and as a consequence is not convex.

presentative portfolios are indeed diversified in the sense that their Diversification Ratio (DR) is never less than that of an EVW portfolio. More precisely,

Property 5: *A maximally ρ-presentative w is fairly diversified as it satisfies*

$$DR(w) \geq DR(w_{evw})/\varrho(w, w_{evw})$$

where w_{evw} denotes the weights of the EVW portfolio and the denominator is the correlation between w and the EVW portfolio (which lies between 0 and 1).

3. Some Well-Known Maximally ρ-presentative Portfolios

While maximally ρ-presentative portfolios may be quite rare, they still unite a striking number of well-known investment strategies.

To illustrate this, let us for a moment wonder how one may build such a portfolio. One simple idea to increase the overall exposure of a portfolio to all assets is to maximize the average of its correlations to the assets:

*The **EVW is maximally ρ-presentative** as it maximizes its average correlation to all the assets amongst all long-short portfolios.*

In other words, the EVW maximizes the average of its spectrum. Once again we observe that we maximize over all - possibly long-short - portfolios and we end up with a long-only portfolio. It is also worth noting that if we consider a large collection of highly correlated assets to which is added another asset sufficiently negatively correlated, the EVW is not necessarily positively exposed to each single asset while being maximally ρ-presentative.

Therefore, we may wonder whether it is possible to build a portfolio that is both positively correlated to each asset and maximally ρ-presentative. For a positive answer, let us focus on portfolios that maximize their minimal exposure:

*The **MDP is maximally ρ-presentative** as it maximizes its minimal correlation to all assets amongst all long-short portfolios.*

Stated otherwise, the MDP maximizes the minimum of its spectrum. Refining this property allows us to prove that amongst long-short portfolios, the MDP also maximizes the minimum correlation to any long-only portfolio. In other words:

The MDP maximizes its lowest exposure to **all long-only factors**, *defined as factors that are replicable by leveraged long-only portfolios of assets belonging to the universe.*

These two latter results also show that *an investor may be positively exposed to the assets* (the minimal correlation in question being always positive) *without necessarily holding them all* (which may happen for the MDP). In contrast, the EVW is invested in all assets without - as explained above - necessarily being positively exposed to each one of them.

Finally we can come up with another maximally ρ-presentative portfolio that is also positively exposed to all assets:

*The ERC is maximally **ρ**-presentative as it maximizes the average of the logarithm of its correlation to all assets.*

Considering Property 5, this implies the following bound on DRs

$$DR(w_{evw}) \leq DR(w_{erc}) \leq DR(w^*)$$

and

$$\varrho(w_{erc}, w_{evw}) \geq \frac{DR(w_{evw})}{DR(w^*)}$$

that reunites all these three remarkable portfolios (namely ERC, EVW and the MDP w*) in an original way. This latter inequality bounds from below the correlation between the ERC portfolio and EVW with the ratio of the DRs of the EVW and that of the MDP.

Before closing this section, the interested reader may consider Table 1 in [1] which provides a unifying framework that includes several other portfolios (*e.g.* the Minimum Variance, the Equal Weight, PCA, *etc*), whereby all strategies maximize an unconstrained objective that is a function of the spectrum ρ(w).

4. Realized Maximal ρ-presentativity and Realized Diversification

In this section, we perform a numerical experiment where we show how the aforementioned bound

$$DR(w) \geq DR(w_{evw})/\varrho(w, w_{evw})$$

satisfied by all maximally ρ-presentative portfolios w can be used to identify actual equity portfolios that qualify for being maximally ρ-presentative without knowing their composition.

Indeed, all the mathematical terms involved in this inequality can be computed using historical time series only. For instance, on the right hand side ϱ(w, w_{evw}) can be measured by simply computing the correlation between the time series of w and that of w_{evw}. The latter is computed thanks to the series of the assets of the universe. Lastly, using time series only, we explained in a previous dashboard how the realized DR of a portfolio with unknown composition can also be measured through the identity

$$DR(w) = DR(\bar{w})\varrho(\bar{w}, w)$$

where $\bar{w}$ is the long-short portfolio with the highest DR. We refer to [1, Proposition 5.3] for further details. In our numerical experiment, we take 464 stocks of the MSCI USA as a universe (having discarded those that did not trade enough over 01/2013 to 03/2017). We similarly considered daily time-series over the same period for 2,278 funds that represent a total of $7500bn *i.e.* 80% of net assets invested in equity in the USA in early 2016 (which also represents half the total net assets).

In Figure 3, we represent the realized DR(w) of these funds as a function of DR(w_{evw})/ϱ(w, w_{evw}). In this figure, *any fund that satisfies this bound is above the dashed line and is candidate for having been maximally ρ-presentative over 2013-2017*. Out of the 2,278 funds considered, one fund, represented by an orange star, satisfies the necessary condition as it lies above the dashed line[1]. Moreover, it reaches the highest realized DR of all 2,278 funds.

[1] as do the forward looking constrained MDPs in green that are indeed maximally ρ-presentative by [1, Proposition 4.15].

This fund is actually the TOBAM Anti-Benchmark® US equity strategy that targets the highest investable DR within the MSCI USA universe.

Thus, the TOBAM US equity strategy preserved the desirable theoretical properties of its parent, the maximally ρ-presentative Most Diversified Portfolio.

As we have seen in the previous section, the EVW and ERC are well-known alternative beta strategies that are maximally ρ-presentative in theory, but none of their implementations appear amongst the portfolios that qualify for being maximally ρ-presentative.

All in all, this experiment shows that:

*Amongst its 2,278 peers, the TOBAM Anti-Benchmark US equity fund has fulfilled its mission in offering the most diversified investable portfolio while being the **only** fund that could claim it has been maximally exposed to all the assets of the MSCI USA over 2013-2017.*

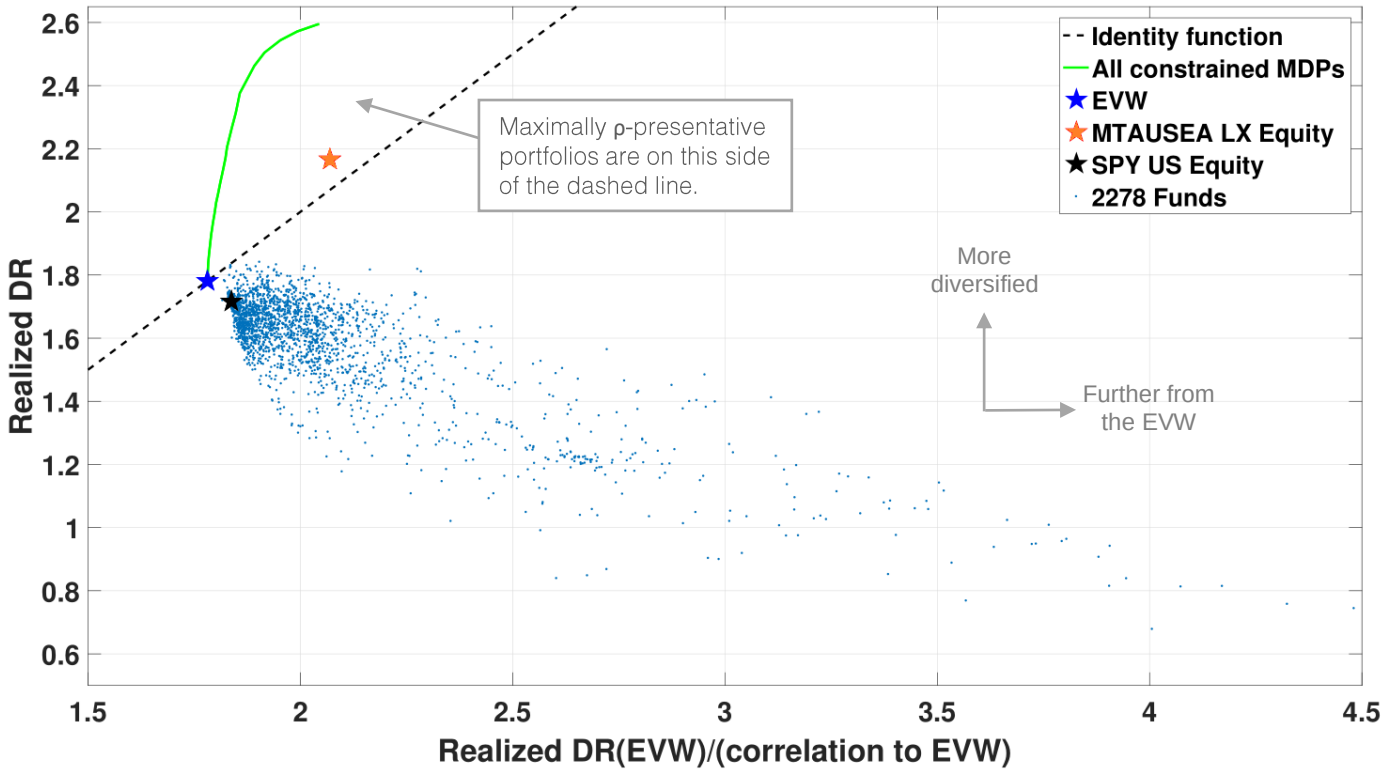

Figure 3: Realized DR and DR(w_{evw})/ϱ(w_{evw},·) in the USA from 01/13 - 03/17 for 2278 funds representing half the total net assets and 80% of the equity assets invested in the USA in Q1/2016. The fund depicted with a blue star is a theoretical and forward looking EVW. The green curve depicts all the forward looking constrained MDPs. This green curve and the dashed line meet precisely at the forward-looking EVW portfolio. The fund in orange is the Anti-Benchmark® US that targets the highest investable DR whereas the fund in black replicates the S&P500. The blue dots depict all other funds. *Only the portfolios that are above the dashed line qualify for being maximally ρ-presentative as they satisfy the necessary condition DR(w) ≥ DR(w_{evw})/ϱ(w,w_{evw})*. The green curve corresponds to theoretical portfolios that are indeed maximally ρ-presentative by [1, Proposition 4.15].

Appendix: Maximally ρ-presentative portfolios of four assets

Considering four different universes of four assets, we depict in Figure 4, the corresponding set of maximally ρ-presentative portfolios.

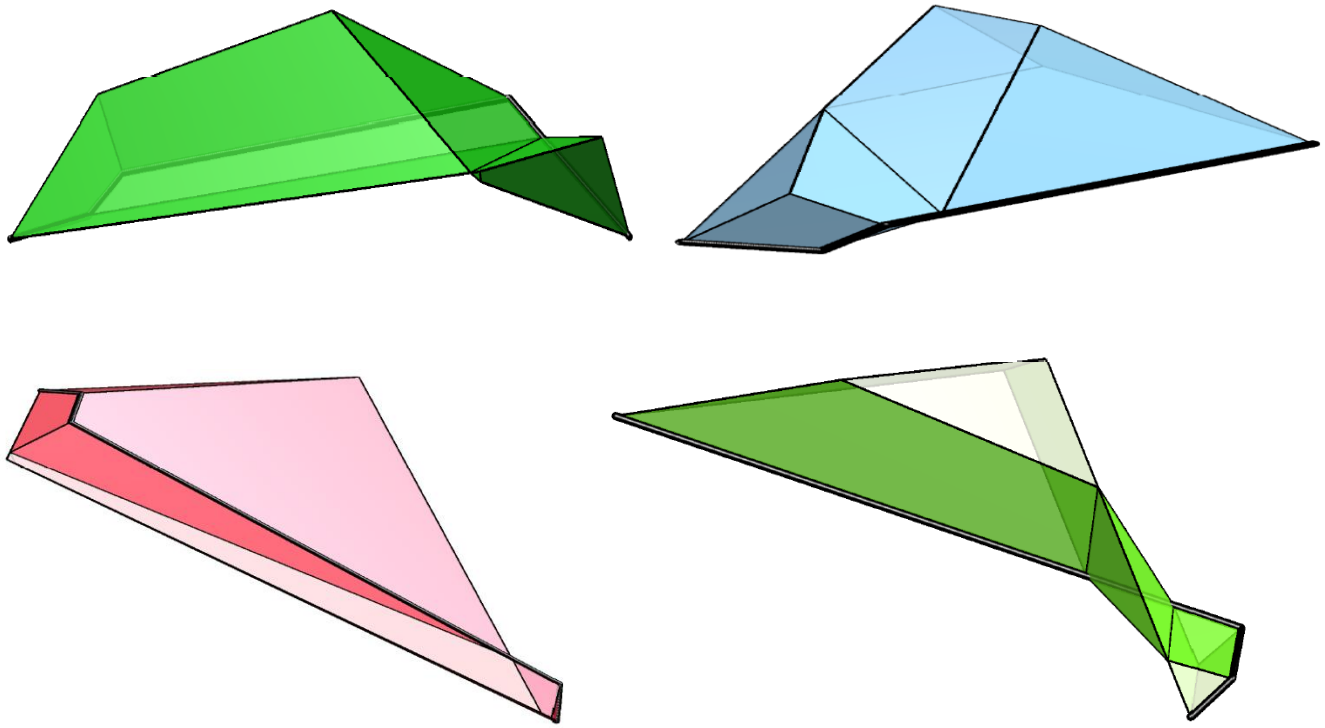

Figure 4: A 3-dimensional representation of the set of maximally ρ-presentative portfolios of four assets for four different universes. The broken thick line represents constrained MDPs that connect the unconstrained MDP to EVW. Observe that each one of these sets is composed of a finite number of polytopes as established in theory in Property 4.

Reference

[1] T. Froidure, K. Jalalzai and Y. Choueifaty, 2017, Portfolio Rho-presentativity, preprint, SSRN.

For more information

TOBAM is an asset management company offering innovative investment capabilities designed to increase diversification. Its mission is to provide rational and professional solutions to long term investors in the context of efficient markets.

The Maximum Diversification® approach, TOBAM's flagship investment process founded in 2006, is supported by original, patented research and a mathematical definition of diversification and provides clients with diversified core exposure, in both the equity and fixed income markets.

In line with its mission statement and commitment to diversification, TOBAM also launched a separate activity on cryptocurrencies in 2017.

TOBAM currently manages over US$10 billion (at June 29, 2018). TOBAM's team is composed of 52 professionals.

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