TOBAM*
CORE INVESTMENTS

DIVERSIFICATION DASHBOARD
October 2015

Diversification Ratios

	Universe	DR^2 Index diversification	DR^2 Maximum diversification	% diversification used by index
TOBAM's Diversification Ratio (DR) measures to what extent a portfolio is diversified. The DR^2 (square of the diversification ratio) measures the number of effective degrees of freedom to which a portfolio is exposed. As the table shows, the "broad market" indices leave diversification on the table. In addition to a snapshot of each market's DR2, the table shows the DR2 of a well-diversified portfolio, and the fraction of available diversification used by the index.	MSCI All Countries World	4.13	12.25	33.7%
	MSCI World	3.66	11.11	32.9%
	MSCI Emerging Markets	5.22	8.85	59.0%
	MSCI Canada	3.62	7.39	49.0%
	MSCI Pacific Ex-Japan	2.43	5.58	43.5%
	MSCI US Equity	2.44	5.30	46.0%
	MSCI Japan	2.00	3.81	52.4%
	MSCI EMU	1.66	3.42	48.6%
	MSCI UK Equity	2.15	3.28	65.5%
	MSCI Australia	1.97	3.25	60.7%

Source: TOBAM, figures as of September 30, 2015

TOBAM will soon celebrate its 10th anniversary. Since TOBAM was founded, our research has been focused on developing a unique knowledge of the Most Diversified Portfolio®, and of its real world avatar: the Anti-Benchmark®. We are pleased to introduce in this dashboard the 35th property of the Most Diversified Portfolio®.

A Property of the Most Diversified Portfolio: the equality of two different risk contribution measures

The asset management industry uses various risk contribution indicators which can lead to different results when interpreting a portfolio risk profile. In this Dashboard, we compare two different approaches to measure the contribution of an asset to the total risk of a portfolio, namely the Risk Contribution and the Average Volatility Contribution. TOBAM theoretical research proves that the Most Diversified Portfolio® (MDP), as opposed to <u>any</u> other long-only portfolio, reconciles both approaches. We present this unique property of the MDP, illustrated through a numerical example. To conclude, we compare the MDP with other long-only portfolios (including some "risk weighted strategies").

1. Definition of the two risk contribution measures and a simple illustrative example

In this Dashboard, we study two common risk contribution measures:

1) Risk Contribution (RC) which can be written in a simple way as:

$$\text{RC of Asset i} = \frac{\text{Weight of asset i in the portfolio x Volatility of asset i x Correlation of asset i to Portfolio}}{\text{Portfolio Volatility}}$$

The RC measures the total risk contribution of an asset to the volatility of the portfolio, in percentage.

2) Average Volatility Contribution (AVC)

$$\text{AVC of Asset } i = \frac{\text{Weight of asset i in the portfolio x Volatility of asset i}}{\text{Weighted average volatility of the Portfolio}}$$

The AVC indicates the contribution of the volatility of one asset to the weighted average volatility of the portfolio, in percentage.

As an aside, we recall that the **Diversification Ratio (DR)** of a portfolio is defined as:

$$\text{DR of the portfolio} = \frac{\text{Weighted average volatility of the Portfolio}}{\text{Portfolio Volatility}}$$

Notice that the denominator of both the RC and the AVC come into play in the definition of the Diversification Ratio.

To get a better understanding of these two risk indicators, let us pick a simple example. Taking the MSCI USA as our chosen universe, we group the assets by sector and then compute the contribution of each sector to the risk of a Minimum Variance portfolio – see Figure 1.

Figure 1: Minimum Variance Portfolio Sector Contribution using both AVC and RC
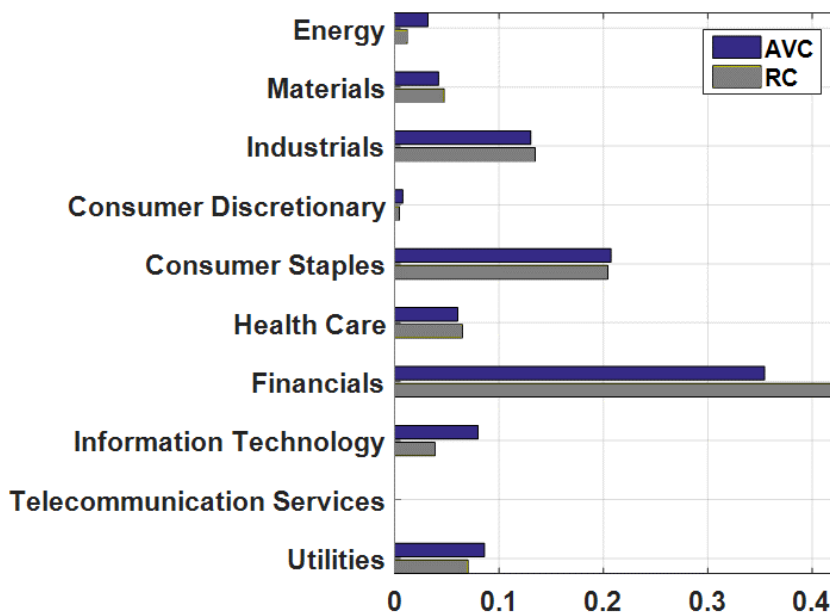
Source: TOBAM. Timeframe from October 2014 to October 2015. Max weight: 6%

Firstly, Figure 1 highlights some of the limitations of this representation: an underweighted sector, such as Telecommunication Services, appears as not contributing to the risk whereas this does not necessarily mean that the portfolio is not exposed to this sector. Think for example of the oil price: not being invested in oil companies does not imply not being exposed to the oil price. In our example, the correlation of the Minimum Variance portfolio to Telecommunication Services (73%) is higher than the correlation to Utilities (58%). In fact, Utilities is the one sector displaying the lowest correlation to the Minimum Variance portfolio.

Back to our main point, this simple example raises an interesting theoretical question: when do these risk contribution indicators coincide? We are going to tackle this question in the forthcoming two sections.

2. A Property of the MDP: equality of Risk Contribution and Average Volatility Contribution

Let us consider an investment universe and define the two risk contribution measures. In the sequel, MDPs can be formed either on the whole universe or on a subset of the assets of the universe. Then, we can prove the two following statements (see *Additional Properties of the Most Diversified Portfolio* in the reference for the formal proof):

1) For any Most Diversified Portfolio, the Risk Contribution of one asset is equal to its Average Volatility Contribution:

Most Diversified Portfolio (formed on a subset or all the assets)	→	Risk Contribution of asset i = Average Volatility Contribution asset i
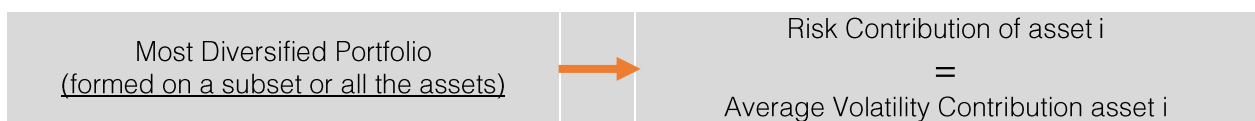
2) Any long-only portfolio, in which the Risk Contribution of one asset is equal to its Average Volatility Contribution, is a Most Diversified Portfolio:

Most Diversified Portfolio (formed on a subset or all the assets)	←	Risk Contribution of asset i = Average Volatility Contribution asset i
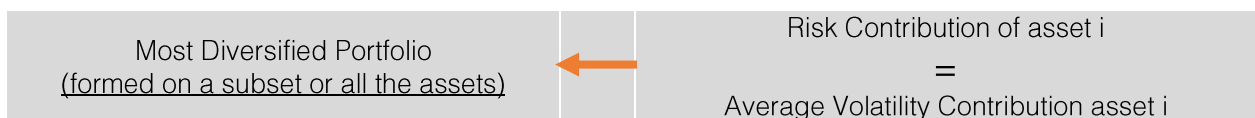
We wish to insist on the fact that this unique equivalence property between the Risk Contribution and the Average Volatility Contribution is valid for any MDP formed on any subset of assets in the universe of investment. To make it clear, this is illustrated in the following figure:

Figure 2: The equivalence holds for any MDP formed either on a subset or on all the assets.
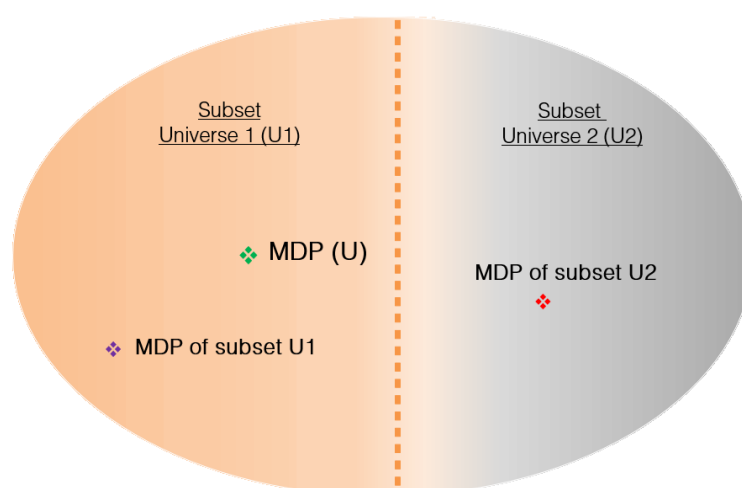
In other words, the previously mentioned mathematical equivalence is valid for all the 3 MDPs formed on their relevant investment universe (subset U1, subset U2, whole universe U) but it is notably also valid for MDP (subset U1) and MDP (subset U2) in the larger combined universe U.

3. Illustration of the property and a comparison with other long-only portfolios

As an illustration, take an investment universe composed of three assets denoted 1, 2 and 3, and let us compare the distance between the Risk Contribution and the Average Volatility Contribution for <u>all</u> long-only portfolios one can form. In particular, we highlight the results for the following risk-weighted strategies:

- Equally-weighted: EW
- Minimum Variance: MV
- Risk Parity: ERC

To be precise, we consider a fixed correlation matrix with
- $\rho_{1,2} = 0.8$,
- $\rho_{1,3} = 0.2$,
- $\rho_{2,3} = 0.3$,

 and volatilities
- $\sigma_1 = 30\%$,
- $\sigma_2 = 40\%$,
- $\sigma_3 = 20\%$.

Given any long-only portfolio w=(w1,w2,w3) in this universe, let us define a distance D as follows :

D(portfolio w)=<u>the distance bewteen AVC and RC</u>.

Note that our main result can be expressed using D, namely

w=MDP <u>(formed on a subset or all the assets)</u>	←→	D(w)=0

which justifies how this quantity may be of interest for us.

In Figure 3, we depict the distance D for all the long-only portfolios of this universe (the MDPs correspond to points that are dark blue that is where D(w)=0).

It should be noted that <u>any long-only portfolio is represented</u> in Figure 3 and is determined in a unique way by the x-axis w1 and y-axis w2 since w3 is obtained by computing 1-w2-w1 and is therefore superfluous in our representation.

Figure 3: Distance D computed for all long-only portfolios of a universe of three assets 1, 2, 3
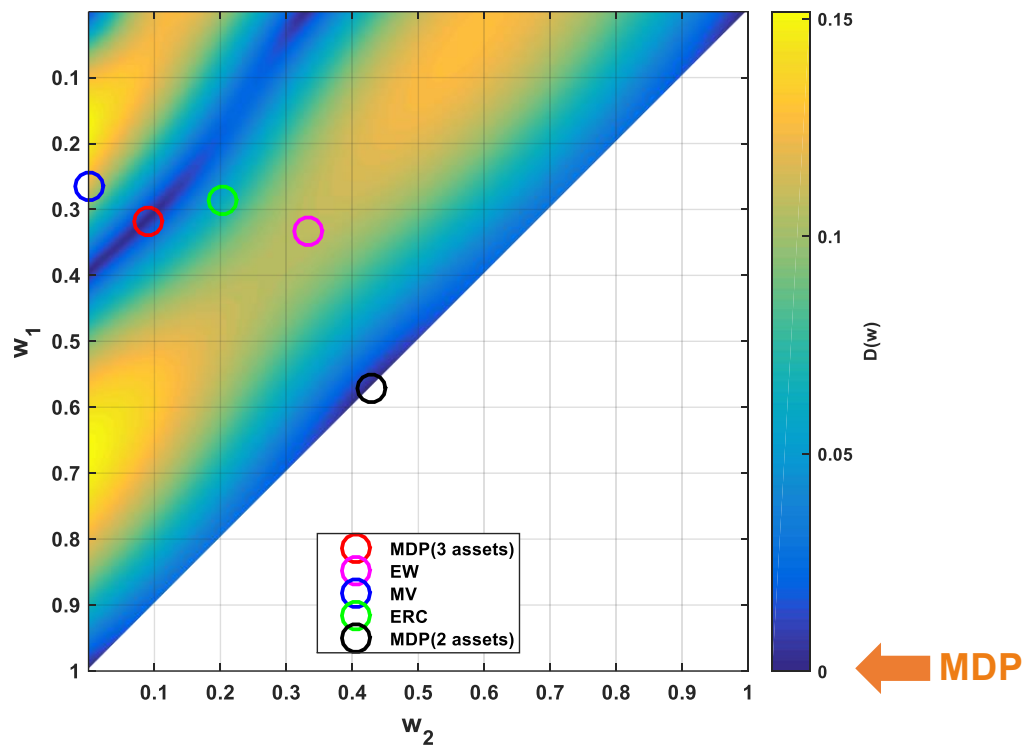
The darkest areas in Figure 3 correspond to portfolios that have risk profiles in terms of RC and AVC that are relatively close. On the other hand, the lighter areas correspond to portfolios with very different risk profiles.

As expected, the distance between the Risk Contribution and the Average Volatility Contribution is zero for both the MDP formed on the whole universe of 3 assets and an MDP formed on a subset of 2 assets. This is however neither the case for the risk-weighted strategies highlighted (excluding the MDP of course), nor for all the other long-only portfolios.

This unique property indicates that the Most Diversified Portfolio risk profile does not depend on the choice between these two risk contribution indicators, as opposed to other long-only strategies.

To complement Figure 3, we report in the following table the figures corresponding to the highlighted portfolios:

Figure 4: RC and AVC of highlighted portfolios, per asset.

Strategy	Contribution Indicator	Asset 1	Asset 2	Asset 3
MDP (3 assets)	AVC	38	15	47
	RC	38	15	47
EW (3 assets)	AVC	33	44	22
	RC	36	51	13
MV (3 assets)	AVC	35	0	65
	RC	26	0	74
ERC (3 assets)	AVC	32	30	38
	RC	33	33	33
an MDP (2 assets)	AVC	50	50	0
	RC	50	50	0

4. Conclusion

We have proven that the Most Diversified Portfolio, formed on any universe or sub-universe of assets, is the only portfolio for which the Risk Contribution of an asset is equal to its Average Volatility Contribution. Aiming to achieve maximum diversification, the Most Diversified Portfolio does not have its risk profile varying according to these two different risk contribution measures. This can be explained intuitively by the construction approach of the MDP, which consists of maximizing the Diversification Ratio, defined as the ratio of the weighted average volatility of the portfolio and the volatility of the overall portfolio.

5. Reference

Tristan Froidure, Khalid Jalalzai, Julien Reynier, *Additional Properties of the Most Diversified Portfolio.* Preprint available at: http://www.tobam.fr/files/AdditionalPropertiesOfTheMDP.pdf

For more information

TOBAM is an asset management company offering innovative investment capabilities whose aim is to maximize diversification. TOBAM's Maximum Diversification® approach, supported by original, patented research and a mathematical definition of diversification, provides clients with diversified core exposure, in both the equity and fixed income markets. The company manages $8 billion (as of September 2015) via its Anti-Benchmark® strategies in Equities and Fixed Income. Its team includes 38 investment professionals.

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