TOBAM
CORE INVESTMENTS

DIVERSIFICATION DASHBOARD

November 2016

Diversification Ratios®

TOBAM's Diversification Ratio® (DR) measures to what extent a portfolio is diversified. The DR^2 (square of the diversification ratio) measures the number of effective degrees of freedom to which a portfolio is exposed.
As the table shows, the "broad market" indices leave diversification on the table. In addition to a snapshot of each market's DR^2, the table shows the DR^2 of a well-diversified portfolio, and the fraction of available diversification used by the index.

Universes	DR^2 Index diversification	DR^2 Maximum diversification®	% diversification used by index
All Countries World	4.28	12.81	33.4%
World	4.06	11.72	34.7%
Canada	3.90	9.42	41.4%
US Equity	2.97	7.43	40.0%
Emerging Markets	3.67	7.29	50.3%
Pacific Ex-Japan	2.23	5.65	39.5%
UK Equity	2.50	3.78	66.1%
Japan	1.85	3.68	50.2%
EMU	1.90	3.50	54.3%
US Credit	3.21	4.25	75.5%

Source: TOBAM, figures as of October 31, 2016

On the Decomposition of the Diversification Ratio

In this dashboard, we provide an in-depth study of the Decomposition of the Diversification Ratio (DR) of a long-only equity portfolio.

As we shall see, this decomposition involves two different measures: the Concentration Ratio of the portfolio and its average correlation.

We will first present theoretical and empirical properties of each measure, and then show that as a result of the DR Decomposition, a diversified portfolio should contain as many assets as possible that are as different as possible. In a nutshell:

In order to build a diversified portfolio, one should put its eggs in baskets that are as numerous and as *different* as possible.

A corollary is that a diversified portfolio is not necessarily invested in all assets available. We provide in the last part of this Dashboard an empirical illustration using US Equities, where we investigate the Maximal DR achievable for a long-only equity portfolio containing a given number of stocks.

1. Herfindahl Index, Effective Number of Stocks & Portfolio Concentration Ratio

Herfindahl Index & Effective Number of Stocks

The Herfindahl index finds several crucial applications in many different fields. In economics, it is known as a measure of the size of companies in a given industry and an indicator of the amount of competition among them. In physics, it allows for distinguishing localized situations in the phase space of disordered systems. From a portfolio manager standpoint, the Herfindahl index quantifies the concentration of weights in an allocation strategy.

The Herfindahl index of a fully invested portfolio without leverage is given by the sum of the squared weight of each asset. More formally, for a portfolio with weights w in a universe of N assets,

$$Herfindahl(w) = \sum_{i=1}^{N} w_i^2.$$

From this definition, we realize that the Herfindahl index actually measures the inverse effective number of assets in a portfolio, which we can rewrite

$$Herfindahl(w) = \frac{1}{N} + N.Variance(w).$$

Indeed, let us consider the following two extreme cases:

- The portfolio for which the Herfindahl index reaches its minimum value $\frac{1}{N}$ is such that its weights have zero variance and are hence all equal: the equal weighted portfolio. In that case the inverse of the Herfindahl is indeed equal to the number of assets in the portfolio.

- The opposite case is when we invest only in one asset, for which we easily see that Herfindahl=1.

To summarize, the inverse of the Herfindahl index interpolates between 1 and N for a single asset portfolio and an equal weighted portfolio, displaying why it embodies the notion of effective number of assets in a portfolio. We can hence compute the effective number of stocks of a portfolio with:

$$Effective\ Number(w) = \frac{1}{\sum_{i=1}^{N} w_i^2}.$$

Portfolio Concentration Ratio

For risk management purposes, we are rather interested in considering risk-adjusted weightings, i.e. where asset weights are also proportional to their risk, which we assume here to be measured by their volatility σ_i. In order to do so, it suffices to consider the following definition of the **risk-adjusted weights**

$$w_i^r = w_i\sigma_i / \sum_{j=1}^{N} w_j\sigma_j$$

to see that all the above arguments on the Herfindahl index can be repeated verbatim. Furthermore, this definition allows us to generalize the former definition of the Herfindahl index since it henceforth captures not only the concentration of weights but also the concentration of risks. As a consequence, we shall refer to this generalized Herfindahl index as the **Concentration Ratio** which reads:

$$CR\ (w) = \frac{\sum_{i=1}^{N}(w_i\sigma_i)^2}{\left(\sum_{i=1}^{N} w_i\sigma_i\right)^2}.$$

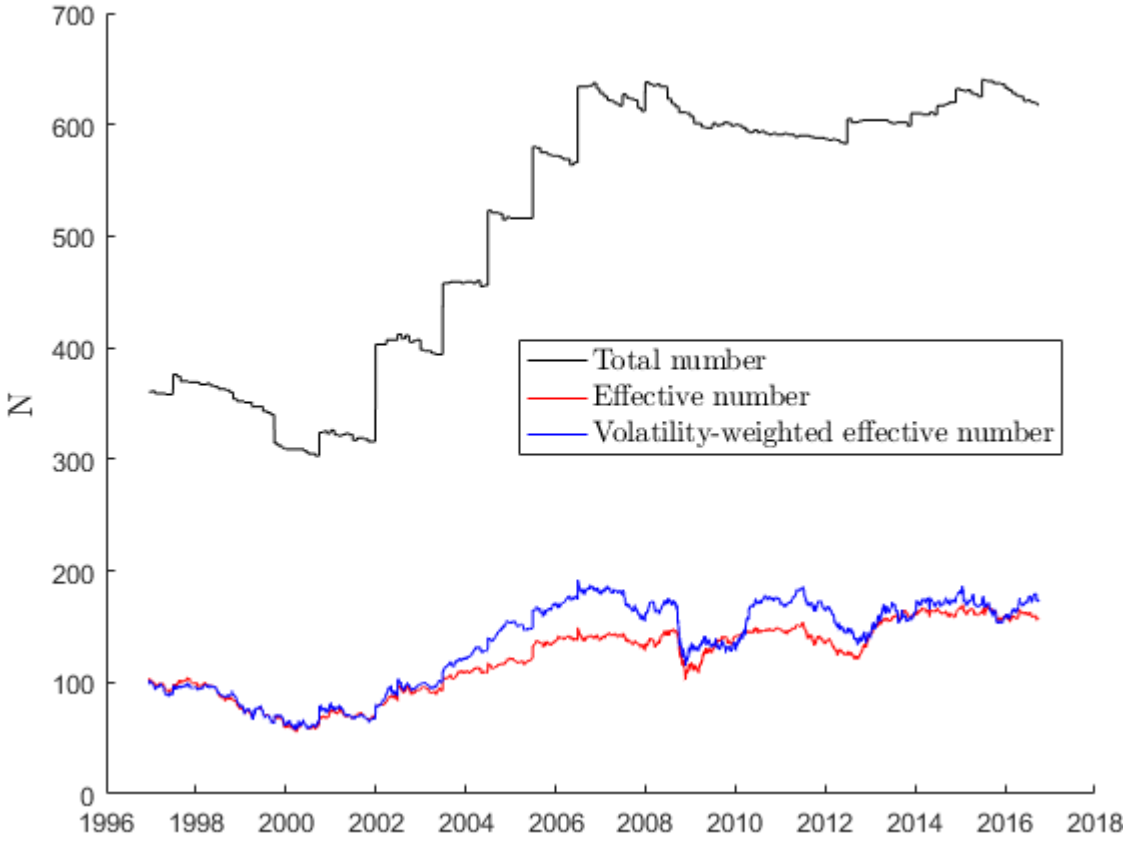
Note that the concentration ratio reaches its minimum value for the equal volatility-weighted (EVW) portfolio with N assets and *not* the equal weighted portfolio.

Now, similarly to the Herfindahl Index, this allows us to define the inverse of the concentration ratio as the **volatility-weighted effective number of assets** of a portfolio.

Empirical Illustrations

The following Figure 1 illustrates the evolution of the effective number of stocks with its volatility-weighted version for the MSCI USA index from 01/01/1997 to 09/29/2016 and compare them against the total number of stocks included in the index.

Figure 1: Number of Assets in the MSCI USA index



Source: TOBAM.

We conclude from Figure 1 that both the effective number of assets in the MSCI USA index (red line) and its volatility weighted counterpart (blue line) were consistently lower compared to the total number of assets included in the MSCI USA (black line). There are two key main takeaways in our view:

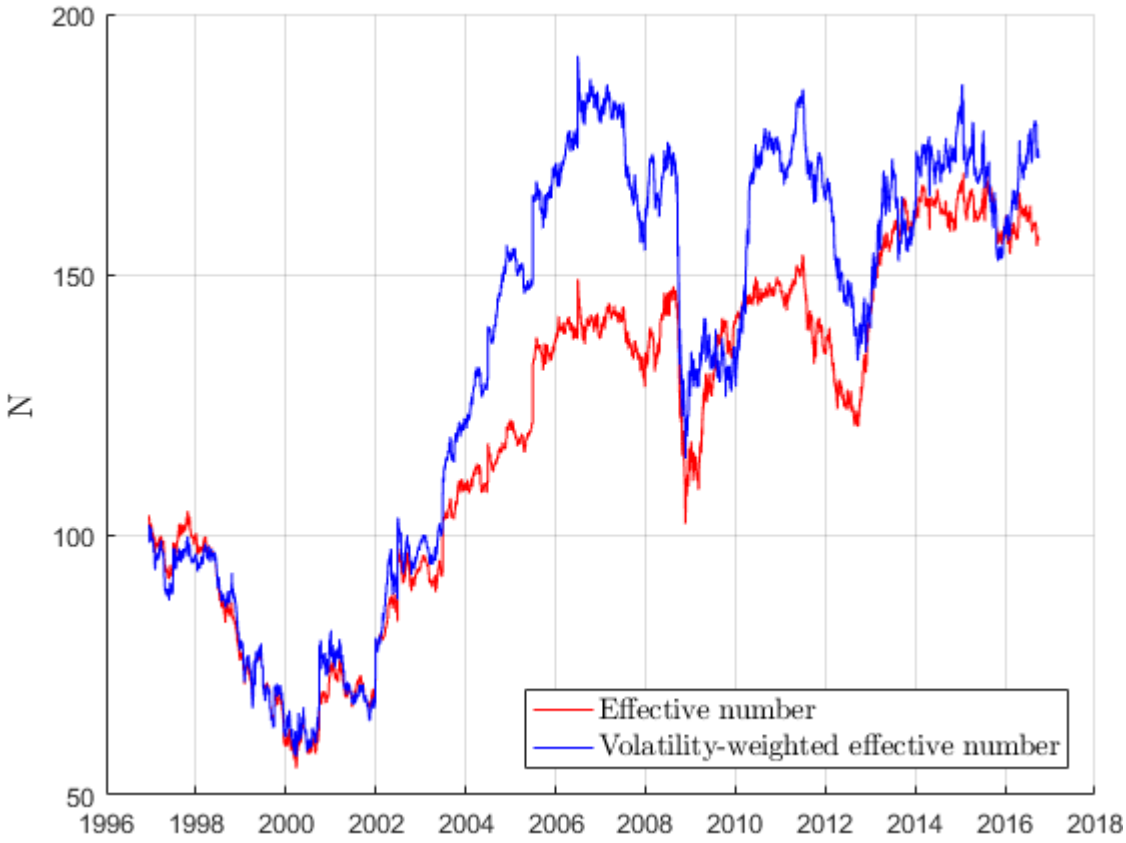
- Since the MSCI USA Index is concentrated in higher market capitalization stocks, it effectively comprises only 80 to 180 stocks for a universe of 300 to 650 stocks.

- Despite the notable increase in the size of the MSCI USA universe from 300 stocks to more than 600 stocks between the end of the internet bubble and the financial crisis, its effective number of stocks has increased at most in proportion, meaning that relative to its current size, it is

roughly as concentrated (as defined by the number of assets) as it used to be before the universe expansion.

The following Figure 2 presents the evolution of the effective number of stocks with its volatility-weighted version for the MSCI USA index from 01/01/1997 to 09/29/2016.

Figure 2: Number of Assets in the MSCI USA index



Source: TOBAM.

Figure 2 illustrates well the high concentration achieved by the MSCI USA during the internet bubble (1997-2000).

It also shows that during other relatively benign market environments, the volatility-weighted effective number of stocks contained in the MSCI USA exceeded its corresponding effective number of stocks. This can be explained by high market capitalization stocks exhibiting a lower volatility compared to smaller market capitalization stocks during these periods (thus producing benchmark risk adjusted weights that are relatively more homogenous than the original benchmark weights).

2. Portfolio Average correlation

Another fundamental input that appears in the DR decomposition is the notion of risk-weighted average correlation, defined formally as

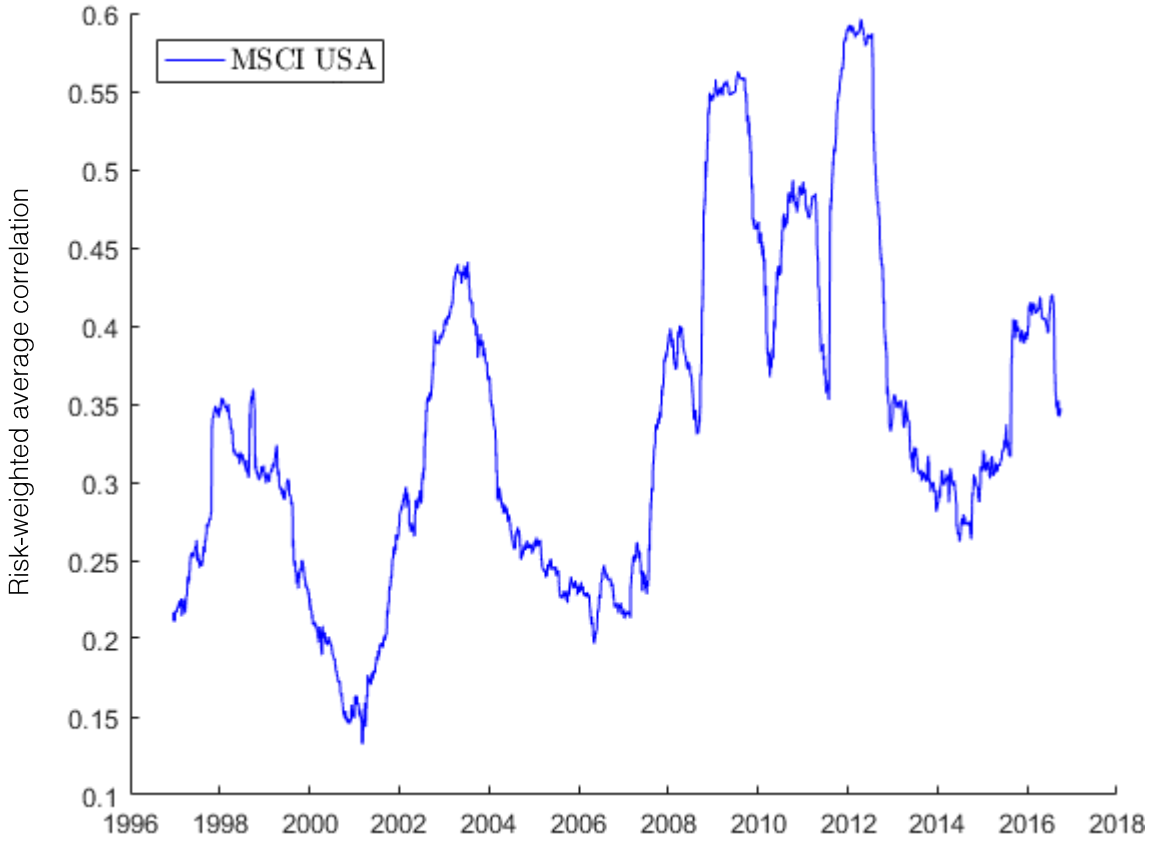
$$\rho(w) = \sum_{i \neq j}^{N} \left[\frac{w_i^r \times w_j^r}{\sum_{i \neq j}^{N} w_i^r \times w_j^r} \right] correlation(i,j)$$

where the correlations between two assets are weighted by their corresponding risk-adjusted weights.

An interesting portfolio in this context is the one that minimizes its average correlation coefficient. Again, if we restrict our study to the set of portfolios which are long-only, fully invested and without leverage, this portfolio is given by the equal volatility-weighted portfolio of the 2 assets that have the lowest pairwise correlation coefficient. It is an easy exercise to find that CR = 1/2 for that specific portfolio.

We illustrate in Figure 3 the risk-weighted average correlation coefficient ρ for the MSCI USA index from 01/01/1997 to 09/29/2016.

Figure 3: Risk-weighted average correlation of the MSCI USA index



Source: TOBAM.

There are a few observations to be made from this graph:

- The risk-weighted average correlation was the lowest around the local peaks of the MSCI USA (2000, 2007, 2015).

- We also note that the risk-weighted average correlation of the MSCI USA was the lowest around the peak of the internet bubble. Indeed, in this particular case the whole market was split across the "Old" vs. "New Economy" divide, with this two "sectors" behaving very differently and thus being lowly correlated. In contrast, Figure 2 shows that the MSCI USA had reached its highest concentration around the same time.

- Naturally high levels of risk-weighted average correlation where reached around the troughs of the market (2002-2003, 2009,2011/2012).

3. On the optimal number of assets to hold in a diversified portfolio

In Choueifaty et al. (2013), the authors showed that the diversification ratio satisfies

$$\frac{1}{\mathrm{DR}(w)^2} = \rho(w) \times \big(1 - \mathrm{CR}(w)\big) + \mathrm{CR}(w)\,.$$

As can be readily inferred from the above decomposition, the fact that the DR can be split into the risk-weighted average correlation coefficient and the risk-weighted concentration ratio formalizes the idea that portfolios with "concentrated" weights and/or highly correlated holdings would be poorly diversified.

Moreover, we get from the DR decomposition that the MDP –the portfolio that maximizes the DR- can in fact be seen as the portfolio that minimizes jointly its Concentration Ratio and risk-weighted average correlation. Indeed:

- For a fixed risk-weighted average correlation coefficient, maximizing the DR reduces to minimize the CR or to maximize the volatility-weighted effective number of stocks in the portfolio.

- For a fixed CR or fixed volatility-weighted effective number of stocks, the MDP is the one that minimizes the risk-weighted average correlation coefficient.

We can now conclude that a well-diversified portfolio as measured by the DR is a portfolio that:

- Invests in as many assets as possible as measured by its volatility-weighted effective number of assets

- Invests in assets that are as different as possible, as measured by their risk-weighted correlations

In a nutshell: in order to build a diversified portfolio, one should put its eggs in baskets that are as numerous and as different as possible.

4. On the optimal number of assets to hold in a diversified portfolio

Let us first recall two results obtained in the first two sections of this Dashboard:

- The portfolio that minimizes its concentration as measured by the CR is the equal volatility-weighted portfolio, or equivalently the portfolio that has the highest volatility weighted effective number of stocks.

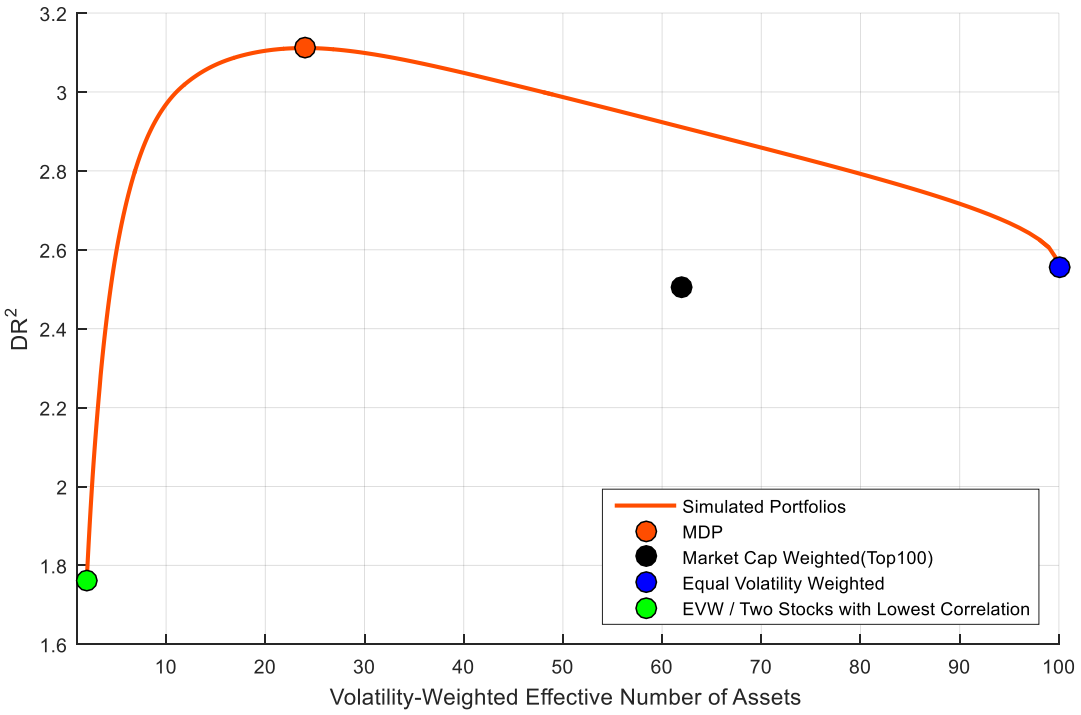
- The portfolio that minimizes its risk-weighted average correlation is the portfolio that invests in the two stocks that have the lowest correlation, with weights inversely proportional to their volatility.

Thanks to the DR decomposition of section III, we can infer that a well-diversified portfolio in a DR sense contains more than 2 assets, and less than the number of assets of the universe it is built on.

We go further in this section and investigate the maximum level of Diversification (as measured by its DR^2) achievable by a portfolio that contains a given number of Stocks (as measured by its volatility weighted effective number of stocks).

Figure 4 below shows the maximal DR^2 that can be achieved by a long-only unlevered portfolio with fixed volatility-weighted effective number of assets, within the top 100 stocks by Market Cap of the MSCI USA as of September 30th, 2016, where the correlation matrix has been shrunk by half towards a constant correlation matrix (for ease of computation of the part of the curve on the left of the MDP).

Figure 4: DR^2 vs. Volatility Weighted Effective Number of Stocks trade-off



Source: TOBAM.

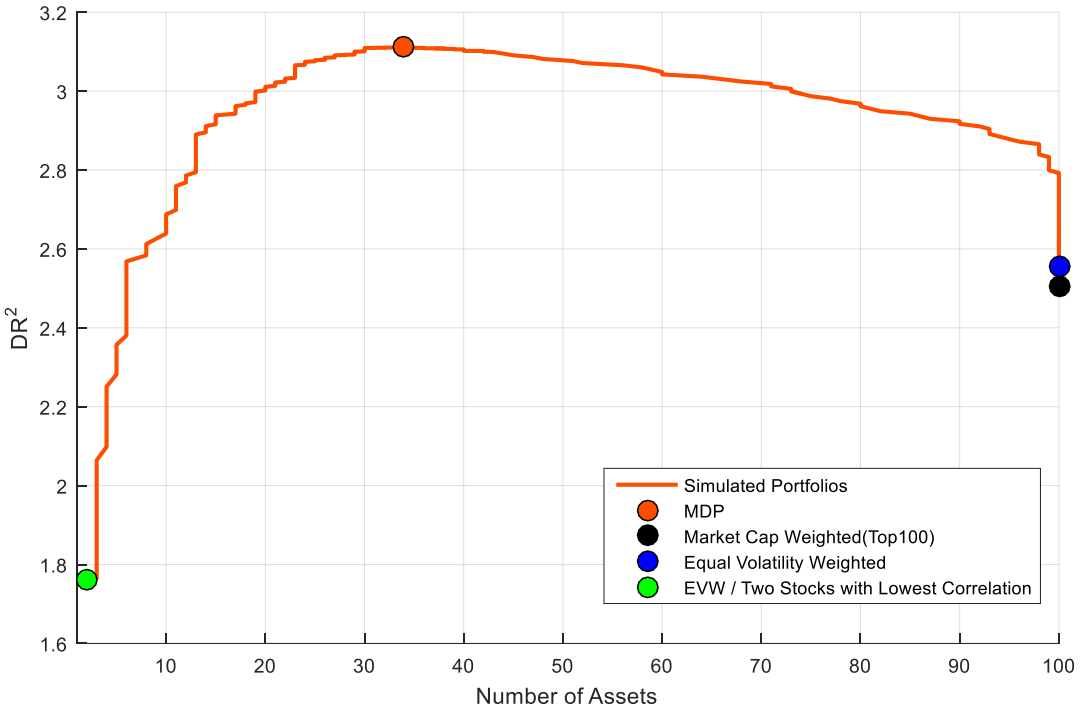
We can conclude from this graph that:

- The market cap-weighted index in this example is marginally less diversified than an equal volatility-weighted portfolio of all asset of the universe, showing its poor diversification characteristics.

- There is an optimal volatility-weighted number of stocks for a portfolio to maximize Diversification, after which Diversification declines. We can infer from the previous section the reason: there is a point where the new assets that are included are too similar. At this stage, increasing the number of assets increases relatively more the portfolio average correlation ρ than it decreases the portfolio concentration CR.

Figure 5 below shows instead the DR^2 of the portfolios of figure 4 versus the number of stock contained in these portfolios (regardless of the magnitude of their associated weights):

Figure 5: DR^2 vs. Number of Stocks



Source: TOBAM.

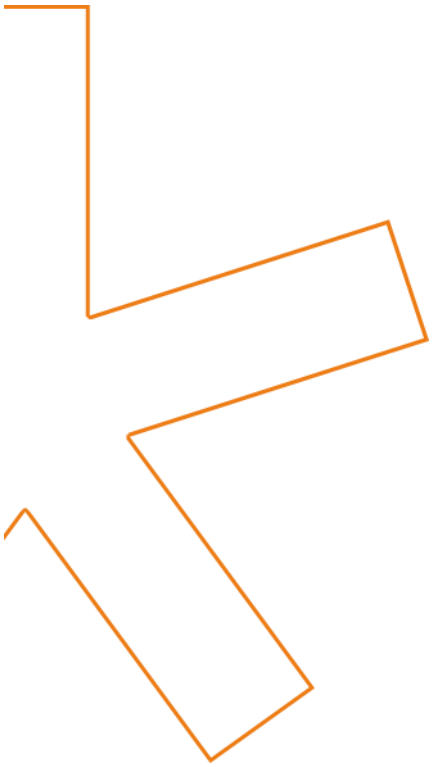
A final conclusion from the above figure is indeed that a diversified portfolio in a DR sense does not invest in all the assets of the investment universe. This result is reminiscent of the work of Statman (1987) where the author gave evidence that for an investable universe of 500 different assets, "a well-diversified portfolio of randomly chosen stocks must include at least 30 stocks for a borrowing investor and 40 stocks for a lending investor".

Even though this very early work was based on different premises and is qualitative in nature, it is consistent with the quantitative results obtained using the DR as a measure of portfolio diversification.

5. References

Yves Choueifaty, Tristan Froidure and Julien Reynier, Properties of the most diversified portfolio, Journal of Investment Strategies, Volume 2/Number 2, Spring 2013 (1–22)

Meir Statman, How many stocks make a diversified portfolio?, Journal of Financial and Quantitative Analysis 22.03, 1987, (353-363).



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