TOBAM
CORE INVESTMENTS

DIVERSIFICATION DASHBOARD

October 2018

Diversification Ratios®

TOBAM's Diversification Ratio® (DR) measures to what extent a portfolio is diversified. The DR^2 (square of the diversification ratio) measures the number of independent sources of risk to which a portfolio is exposed.
As the table shows, the "broad market" indices do not fully utilise diversification capabilities. In addition to a snapshot of each market's DR^2, the table shows the DR^2 of a well-diversified portfolio, and the fraction of available diversification used by the index.

Universes	DR^2 Index diversification	DR^2 Maximum Diversification®	% diversification used by index
MSCI All Countries World	4.97	17.06	29.1%
MSCI World	4.72	15.36	30.7%
MSCI Canada	4.81	15.18	31.7%
MSCI US Equity	4.00	11.60	34.5%
MSCI Emerging Markets	4.30	9.43	45.6%
MSCI Pacific Ex-Japan	3.87	9.12	42.4%
MSCI UK Equity	3.70	6.51	56.9%
MSCI EMU	3.44	8.69	39.6%
MSCI Japan	3.21	6.74	47.6%
BofA Merrill Lynch US Corporate & High Yield	5.13	7.05	72.7%
BofA Merrill Lynch Global High Yield	7.71	9.66	79.8%

Source: TOBAM, figures as of September 28, 2018.

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Benchmark performance analysis

In this month's Diversification Dashboard, we introduce an innovative approach to analyse the performance of market cap-weighted portfolios.

The starting point of the approach is the fact that these benchmarks do not allocate risk neutrally and are biased as a result. For example, they are biased towards assets that have higher a higher capitalization, by construction. As a result, we propose to analyse performance of the benchmark against a portfolio that allocates risk neutrally, defined as the portfolio that maximizes diversification.

This approach identifies the biases of the benchmark, and then analyses whether these biases have been rewarded - or not - over a given period. Then, the value-added - or detracted - by each of the identified biases is computed, thus providing an explanation of the relative performance of the benchmark versus a portfolio that maximizes diversification.

In the second part of this Dashboard we discuss some common equity factors that might be used in order to identify the biases of market cap-weighted benchmarks.

In the last part of the dashboard, we provide an illustration of the benchmark performance analysis using the example of the MSCI Emerging Market equity Index over the first half of 2018, using the well-diversified Anti-Benchmark EM equity portfolio as reference.

We believe that this approach provides a new and interesting framework for explaining the performance of any buy and hold long-only portfolios, and might be particularly timely for analysing market cap-weighted benchmarks that are currently highly concentrated.

Methodology Description

The goal is to measure the performance of the market cap-weighted benchmark (the "Benchmark") against a portfolio that allocates risk neutrally, defined as the portfolio that maximizes diversification.

- By maximizing diversification, the Most Diversified Portfolio ("MDP")® is the portfolio whose risk is contributed homogeneously by all the risk factors available in the investment universe and allocates risk neutrally. As such, it can be defined as the portfolio without bets
- Any portfolio different from the MDP can be defined as a portfolio with implicit bets.
- The performance of any portfolio, including the Benchmark, can thus be analyzed by looking at the performance of its implicit bets.

We will proceed in three separate steps, and for the sake of clarity we will refer to the chosen portfolio as the "Benchmark" and the reference portfolio that maximizes diversification as the "MDP".

Step 1: Identifying the biases of the Benchmark

In order to identify the biases of the Benchmark, we compute the exposures of the Benchmark and of the Reference Portfolio to a set of predefined factors separately. The difference in exposures then allows to identify the biases of the Benchmark. For example, if the Benchmark has a positive exposure to a given factor while the Reference Portfolio has a lower exposure to this factor, we will then conclude that the Benchmark is positively biased towards the given factor.

Since the biases of the Benchmark Portfolio might come from many different sources, we will need to consider a set of many different predefined factors. We will then need to be able to compare the exposures of our portfolios to these factors. In order to do so, we need a measure of exposure that is normalized across these factors, and cannot use for example the betas of our portfolios to these factors (the factor might have different volatilities).

A natural choice in this case is to use correlation as a measure of exposure, which leads us to define the difference in exposures as the difference between the correlation of the Benchmark to a given factor and the correlation of the MDP to the same given factor:

$$\Delta_{BM-MDP} = \rho(BM, F) - \rho(MDP, F)$$

where we denote **BM** the Benchmark, **MDP** the Most Diversified Portfolio and **F** the given factor.

Step 2: Computing the Value-Added by the Benchmark

The Value Added by the Benchmark is defined as its return relative to the beta-adjusted return of the MDP:

$$Value\ Added(BM) = R_{BM} - \beta_{BM/MDP} \times R_{MDP}$$

where R_M, R_{MDP} denote the returns of the Benchmark and of the MDP and by $\beta_{M/MDP}$ the beta of the Benchmark with respect to the MDP.

Note that the Value Added of the Benchmark is the "Alpha" of the Benchmark with respect to the Most Diversified Portfolio. We have refrained however to use this term as it is usually reserved for the reverse expression $R_{MDP} - \beta_{MDP/BM} \times R_{BM}$, which is different, in particular from the opposite of the Value Added by the Benchmark.

Step 3: Computing the Factor contribution to the Value Added by the Benchmark

Next, for each given factor F we decompose the Value Added by the Benchmark in two parts: one that is uncorrelated to the Value Added by the Factor F and thus irrelevant, denoted ε - and the other which we define as the Impact of the Factor F on the Value Added by the Benchmark:

$$Value\ Added(BM) = \gamma_F \times Value\ Added(F) + \varepsilon \quad = Impact(F) + \varepsilon$$

where γ_F is obtained by linear regression (as explained below). As such the impact of the Factor F on the Value Added by the Benchmark is:

$$Impact(F) = \gamma_F \times \left(R_F - \beta_{F/MDP} \times R_{MDP}\right).$$

The above relationship allows one to extract the contribution to the Value Added by the Benchmark of any factor, or impact of any factor on the Value Added by the Benchmark.

Note that the above decomposition and its associated coefficient of determination (R2) could be obtained using a linear regression between the Value-Added by the Benchmark and that of the considered factor F. However, the coefficient γ_F and its associated R2 can be computed directly[1]:

$$\gamma_F = \frac{\rho_{F,BM} - \rho_{F,MDP} \times \rho_{MDP,BM}}{1 - \rho_{F,MDP}^2} \times \frac{\sigma_B}{\sigma_F}$$

$$R^2 = \frac{(\rho_{F,BM} - \rho_{F,MDP} \times \rho_{MDP,BM})^2}{(1 - \rho_{F,MDP}^2)(1 - \rho_{MDP,BM}^2)}$$

With the R^2 measuring the "strength" of the Value-Added decomposition using Factor F, or the bias of the Benchmark towards Factor F, given the information provided by the MDP.

Of note, as the impact of each factor is computed independently, the sum of all the impacts do not sum to the Value Added by the Benchmark. This is understandable as the choice of the factors is arbitrary.

This method is fast, straightforward and allows to use a large number of different factors in order to identify the biases of the benchmark, and their contribution to the Value Added by the Benchmark.

[1] Gamma is in fact the partial correlation between B and F controlled by MDP, scaled by the "'partial volatilities" of B and F. The partial squared correlation – simply denoted R^2 here - quantifies the proportion of the variance of B explained by F, conditional on MDP.

I. Equity Factor Choices

Magnitude of the Factor Impact

In order to choose the relevant factor, we note that the Factor Impact on the Value-Added by the Benchmark is greater *in magnitude* if (all else being equal):

- the Value-Added by the factor is greater.
- the greater the correlation between the Benchmark and the Factor (assuming It is greater than$\rho_{BM,MDP} \times \rho_{F,MDP}$, as would usually be the case).
- the volatility of the factor F is lower or the volatility of the benchmark portfolio is greater.
- the explanatory power of the factor is higher if the R^2 is higher.

Equity Factor Choice Considerations

While any factors can be chosen, we have chosen to use the following publicly available MSCI sectors and factors as they are appropriate for a long only universe, easily obtainable and accepted:

MSCI sectors:
Split into the 11 GICS MSCI sectors: Energy, Materials, Industrials, Consumer Discretionary, Consumer Staples, Healthcare, Financials, Information Technology, Telecommunication Services, Utilities, Real Estate. All the sector indices are market cap weighted and as such, they are all biased towards large cap stocks.

MSCI factors:

- The "Value" factor is invested in cheap stocks relative to their fundamental value.
- The "Growth" factor is invested in stocks that offer superior profit growth expectation. Interestingly, the returns of the "Value" and the "Growth" portfolio sum in twice that of the Benchmark.
- The "Low Risk" factor is invested in the lowest volatility stocks.
- The "Min Vol" factor is invested in an optimized portfolio that combines volatility and correlation to produce a low volatility portfolio.
- The "Large Size" Factor is invested in the largest stocks from a market cap point of view.
- The "Mid-Size" Factor is invested in the smallest stocks from a market cap point of view.
- According to MSCI, this represents the Mid-cap segment. The merging of the "Large Size" and the "Mid-Size" portfolios results in the Benchmark.
- The "Equal Weight" factor weights equally the universe constituents at each rebalancing date.
- The "High Momentum" factor is invested in the best performing stocks in the recent past.
- The "Quality" factor is invested in stocks with durable business models and sustainable competitive advantages.

II. Application: 2018 H1 MSCI Emerging Markets performance

We now consider the application of our approach to the MSCI Emerging Markets during the first half of 2018. We choose in this case the Anti-Benchmark EM as reference portfolio, as it is a well-diversified long-only portfolio, obtained as the result of maximizing the Diversification Ratio.

Tables description and use

The following two tables provide an illustration of the approach. The **"Correlation" tab** shows the correlation of each Factor to the Benchmark and to the Anti-Benchmark, as well as the difference between the two. This tab aims at to measuring in correlation terms the magnitude of the biases of the Benchmark over the period considered.

The **"Performance" tab** shows the return of each Factor, their excess returns vs. the Anti-Benchmark, as well as their volatility.

This tab provides insight to the factors that have significantly outperformed or underperformed the well-diversified portfolio, and towards which the Benchmark is biased, based on the "Correlation" tab. As such, this tab highlights the factors that explain the over/under performance of the Benchmark vs. the Anti-Benchmark.

Finally, **the "Benchmark Performance Explanation" tab** provides the Value Added by each factor over the period considered and their impact on the overall Value Added by the Benchmark with associated R^2s.

To use this tab we select the factors that have the highest R^2, which is an alternative measure to identify the potential biases of the benchmark. The factors that best explain the performance will have a high R^2 and an Impact on the Benchmark Value Added that is close to that of the Benchmark.

Table 1: H1 2018 MSCI EM vs. AB EM performance analysis using MSCI Sectors.

MSCI EM Universe From 29/12/17 to 29/06/18		Average Correlation	BM	AB	Energy	Materials	Industrials	Con. Disc.	Con. Staples	Health Care	Financials	Info Tech	Telecomm	Utilities	Real Estate
Correlation	To BM	83%	100%	89%	75%	88%	92%	90%	83%	57%	94%	93%	83%	81%	79%
	To AB	77%	89%	100%	66%	78%	88%	79%	80%	67%	83%	80%	76%	79%	79%
	Δ BM-AB				9.7%	9.7%	3.7%	10.8%	2.7%	-9.5%	12.0%	12.8%	7.3%	1.7%	0.1%
Performances	Return		-6.7%	-2.7%	2.4%	-4.8%	-11.9%	-12.8%	-6.9%	1.6%	-9.1%	-3.1%	-13.0%	-5.4%	-13.1%
	Return-AB		-4.0%		5.1%	-2.2%	-9.3%	-10.2%	-4.3%	4.3%	-6.4%	-0.4%	-10.4%	-2.7%	-10.5%
	Annualized Volatility		15.1%	12.0%	17.8%	16.1%	14.4%	18.1%	12.1%	21.5%	16.1%	19.6%	12.0%	12.7%	23.7%
Benchmark Performance Explanation	Benchmark Value Added		-3.7%		5.0%	-2.0%	-9.1%	-9.7%	-4.8%	4.8%	-6.1%	0.4%	-11.0%	-3.1%	-9.0%
	Impact on Benchmark Value Added		-3.7%		1.3%	-0.9%	-5.5%	-4.1%	-1.9%	-0.1%	-3.8%	0.2%	-5.0%	-1.0%	-1.3%
	R^2		80%		24%	41%	37%	49%	17%	1%	67%	62%	27%	14%	9%

Negative figure means excess return of AB

Negative figure means excess return for AB

Sources: TOBAM, Bloomberg. Warning: Past performance is not an indicator or a guarantee of future performance. The value of your investment and income received from it can go down as well as up and you may not get back the full amount invested. Performance details provided are in USD and may include reinvested dividends. Performance returns and/or charts illustrating performance provided on this page are Gross of management fees, sales charges and other commissions, other taxes and relevant costs to be paid by an investor are not included in the calculations. The net performance over the period (including management, administrative & subscription fee) would be -3.14%.

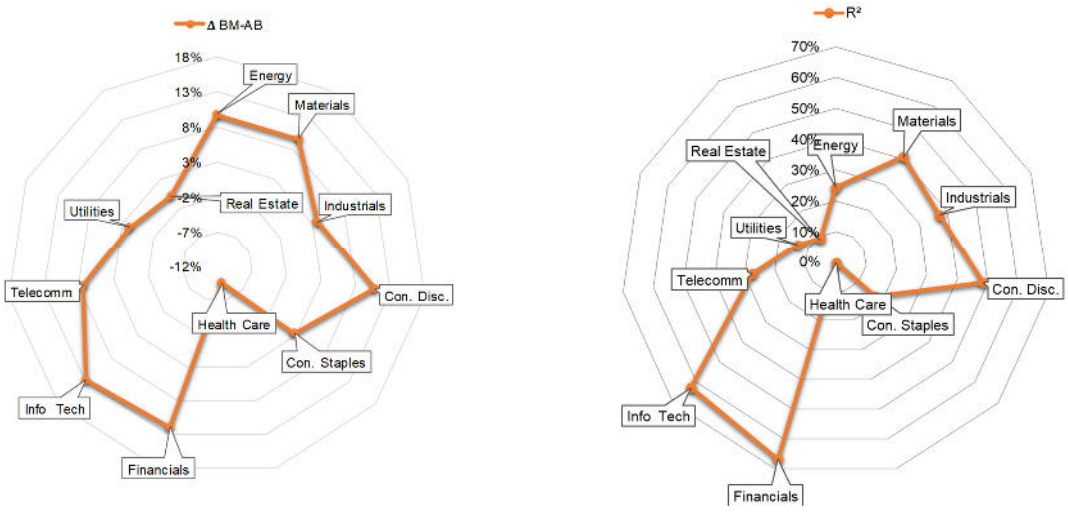
Table 1 provides a specific analysis of the relative performance of the the MSCI Emerging Markets vs. the Anti-Benchmark EM over 2017, using Sectors as potential biases of the Benchmark.

The **"Correlation"** tab shows that the Benchmark had a higher correlation to all sectors with the exception of Healthcare, and had significant biases to cyclical sectors such as Financials, IT, Consumer Discretionary, Materials and Energy as measured by Δ_{BM-AB} row in Table 1.

The **"Performance"** tab shows that amongst the two sectors with the highest positive biases (in orange), the Financial sector underperformed significantly the Anti-Benchmark by 6.4%.

The "**Benchmark Performance Explanation"** tab shows that the Financial sector was one of the main contributors to the under-performance of the Benchmark over the period. The Financial and IT sectors both stand out as potential Benchmark biases as measured by the R^2 – or partial correlation squared, as shown by the second radar plot of Figure 1, were these two particular sectors stand out even more than when using correlation difference as in the first radar plot:

Figure 1: H1 2018 MSCI EM Potential Sector Biases



Source: TOBAM, Bloomberg.

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Amongst the two sectors with the highest R^2 (in orange in the table), Financials stand out with an impact on the value added by the benchmark of -3.8%, which is very close to the full Value Added by the Benchmark of -3.7%. As such, the positive financial bias of the Benchmark was not rewarded over the period and explains most of its underperformance.

The outperformance was not generated via additional risk as the Benchmark was 25% more volatile over the period compared to the Anti-Benchmark, (15.1% vs. 12%).

The following table provides a similar relative performance analysis, using MSCI factors.

Table 2: H1 2018 MSCI EM vs. AB EM performance analysis using MSCI Factors.

MSCI EM Universe From 29/12/17 to 29/06/18		Average Correlation	BM	AB	Value	Growth	Low Risk	Min Vol	Large Size	Mid Size	Equal Weight	High Momentum	Quality
Correlation	To BM	96%	100%	89%	97%	98%	94%	95%	100%	96%	96%	97%	96%
	To AB	90%	89%	100.0%	89%	86%	90%	93%	88%	92%	93%	89%	91%
	Δ BM-AB				8.4%	12.2%	4.4%	2.2%	11.8%	3.1%	2.8%	7.3%	4.7%
Performances	Return		-6.7%	-2.7%	-7.5%	-5.9%	-8.8%	-3.3%	-6.8%	-6.2%	-10.2%	-5.0%	-8.0%
	Return-AB		-4.0%		-4.8%	-3.2%	-6.2%	-0.7%	-4.1%	-3.6%	-7.6%	-2.3%	-5.4%
	Annualized Volatility		15.1%	12.0%	15%	16.3%	11.7%	11.0%	15.5%	13.7%	13.1%	17.4%	13.2%
Benchmark Performance Explanation	Benchmark Value Added		-3.7%		-4.6%	-2.8%	-6.5%	-1.1%	-3.8%	-3.4%	-7.5%	-1.5%	-5.4%
	Impact on Benchmark Value Added		-3.7%		-4.1%	-2.1%	-6.2%	-1.3%	-3.5%	-3.4%	-8.2%	-1.1%	-5.0%
	R^2		80%		77%	86%	51%	53%	99%	58%	59%	70%	59%

Sources: TOBAM, Bloomberg. Warning: Past performance is not an indicator or a guarantee of future performance. The value of your investment and income received from it can go down as well as up and you may not get back the full amount invested. Performance details provided are in USD and may include reinvested dividends.
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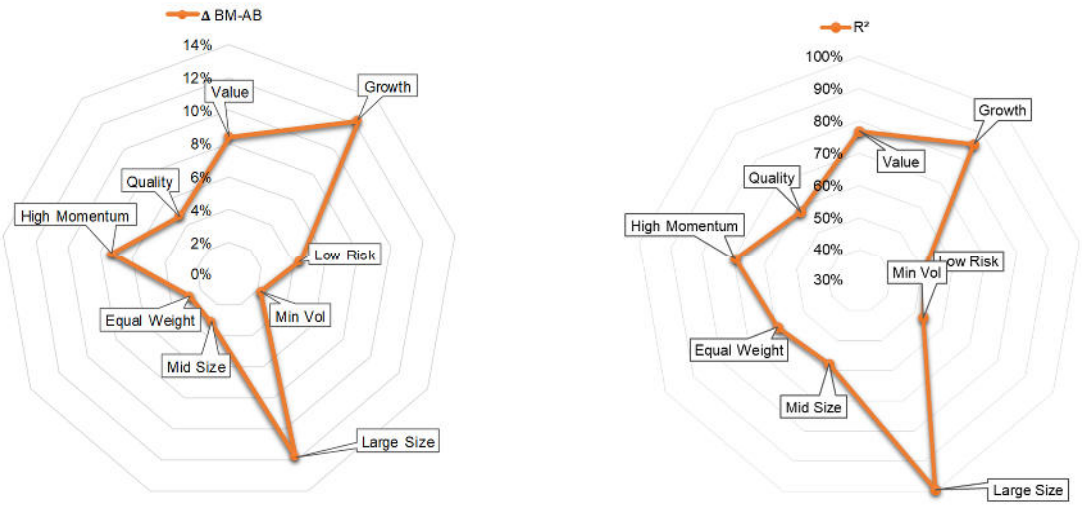
The **"Correlation"** tab shows that Benchmark had significant biases in the Large Size and Growth factors as measured by Δ_{BM-AB} (in orange). As noted earlier, the Large Size bias is expected as the Benchmark exhibits such a bias by construction, relative to a well-diversified portfolio such as the Anti-Benchmark. Figure 2 is an illustration of these potential biases in terms of Correlation differences and R^2.

Figure 2: H1 2018 MSCI EM Potential Factor Biases



Source: TOBAM, Bloomberg.

The **"Performance"** tab shows that the Large Size and Growth factors have significantly under-performed over the period, with an underperformance very close to that of the Benchmark relative to the Anti-Benchmark.

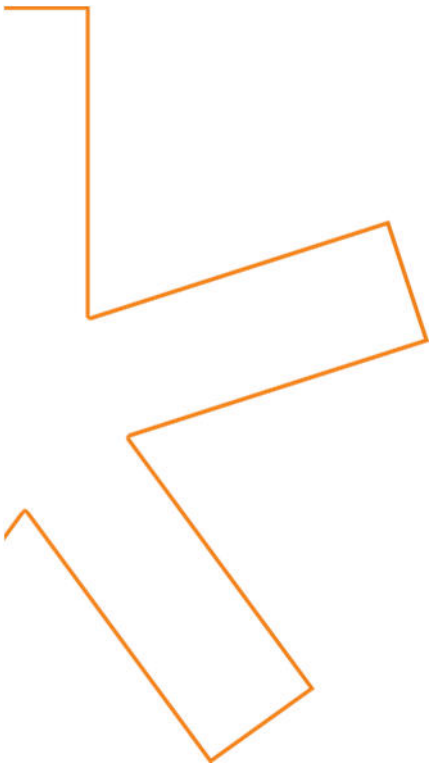
The "**Benchmark Performance Explanation**" tab shows the impact of the Large Size Factor of -3.5% (in orange) is also very close to the full Value Added by the Benchmark of -3.7%. As such, the positive Large Size bias of the Benchmark was not rewarded over the period and also explains most of its underperformance in conjunction to its bias towards Financials.

Conclusion

In this month's Diversification Dashboard, we introduce a new approach to explain the performance of any given portfolio relative to a well-diversified portfolio. This method first identifies the biases of the chosen portfolio, and the impact of each of these biases on its overall Value Added. As a result, the method measures to which extent the biases of the chosen portfolio have been rewarded -or not- over the period of interest.

This new method has the following advantages:

- It uses correlation, which can be measured using times-series only. As a result, sectors and factors compositions are not required and publicly available time-series produced by any index providers can be used.
- The method allows for identifying biases of the chosen portfolio in a systematic manner.
- As opposed to a classic Brinson performance attribution, it does not require a non-overlapping breakdown of the investment universe, such as economic sectors. As such, it can be used with any given set of risk drivers.



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