DIVERSIFICATION DASHBOARD

July 2021

Ergodicity, Diversification and Bitcoin

In this month's Diversification Dashboard, we would like to shed some light on work currently being done at the London Mathematical Laboratory (LML) which has no other ambition than rebuilding modern economics from the foundation up; moreover, we highlight how these very fundamental mathematical and economic principles relate to our interest in Bitcoin.

In this dashboard, we look deeper into:

1- Ensemble vs Time averages

- How the concept of Expected Value can be misleading
- The lack of ergodicity of most financial processes or the paradox of Russian roulette

2- Maximising the expected wealth growth rate

- Maximising the long-term wealth growth rate with the *Kelly criterion*
- The issue of probability-weighted average

3- Why diversification is the solution

- Why low vol isn't low risk
- Bitcoin as a diversifier

Introduction: Russian roulette derived teachings

Randomness and risk are traditionally handled by considering that a random event is a crossroad to many possible futures, one of which we will inhabit. As we cannot know which one road we will take, we value the crossroad by aggregating the desirability – or "utility" – of all the roads it leads to. This is called the *ensemble-average view of decision making* because we value a decision by looking at the ensemble of futures it may lead to at a given date. This familiar view originates from the works of Fermat and Pascal, who introduced the concept of Expected Value (EV) in order to answer the following question "Should a game of dice be interrupted midcourse, what settlement between the partaking gamblers would be fair?" the conclusion being that we should average the Profit and Losses (PnLs) of the gamblers in all possible states of the universe, had the game been brought to its end.[1]

Another view is however possible, it consists in considering what will arrive in the long run if we keep making the same decision over and over again. Here, randomness is sorted out by averaging through time instead of averaging through the ensemble of possible futures at a given date. Interestingly enough, considering what happens when a given decision, or decision-pattern (a strategy), is repeated an infinite number of times is a leitmotif of non-mathematical decision theory, aka morality; see for instance this extract of Nietzsche's Joyful Wisdom [1] (emphasis ours):

> *The heaviest burden.—What, if some day or night a demon were to steal after you into your loneliest loneliness and say to you: "This life as you now live it and have lived it, you will have to live once more and innumerable times more; and there will be nothing new in it, but every pain and every joy and every thought and sigh and everything unutterably small or great in your life will have to return to you, all in the same succession and sequence—even this spider and this moonlight between the trees, and even this moment and I myself. The eternal hourglass of existence is turned upside down again and again, and you with it, speck of dust!"*
>
> *Would you not throw yourself down and gnash your teeth and curse the demon who spoke thus? Or have you once experienced a tremendous moment when you would have answered him: "You are a god and never have I heard anything more divine." If this thought gained possession of you, it would change you as you are or perhaps crush you. The question in each and every thing, "*Do you desire this once more and innumerable times more*?" would lie upon your actions as the greatest weight. Or how well disposed would you have to become to yourself and to life to crave nothing more fervently than this ultimate eternal confirmation and seal?*

The idea of eternal recurrence itself has actually far more ancient roots as it can be traced to Mesopotamian philosophy [2].

Practically, as no two situations are the same in life, the eternal recurrence principle may be more easily understood when applied to our decision-making processes: Instead of evaluating a single decision, assess if the thought-process from which it stems can satisfactorily be applied an infinite number of times.

[1] *Œuvres de Blaise Pascal*, Volume 2, Lefèvre, 1819 (French)

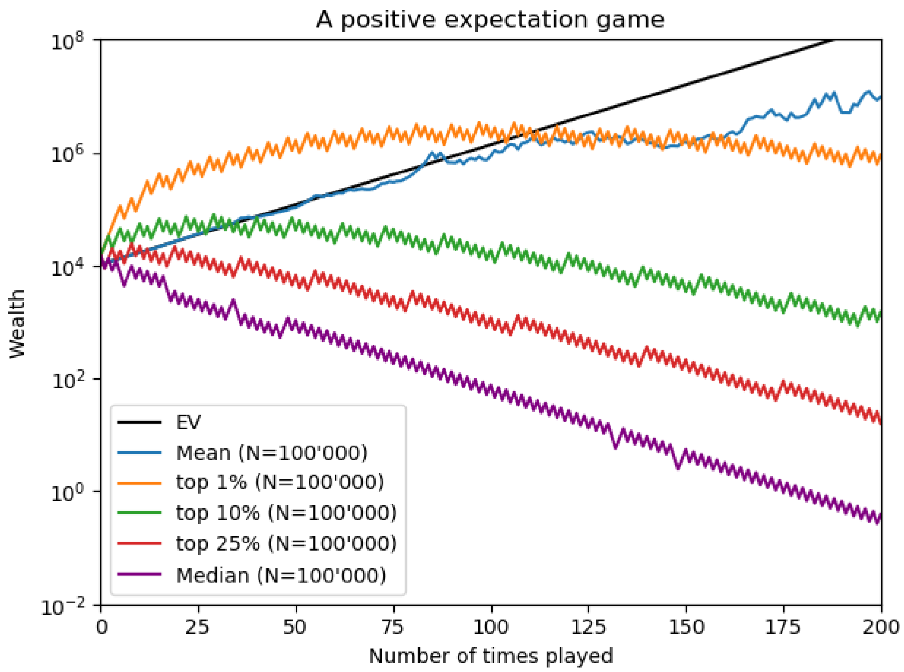
Ensemble vs Time averages

How the concept of Expected Value can be misleading

The point is that ensemble and time views may lead to contradictory results. Let us for instance consider the example of a repeated investment game offering at each iteration a 50% chance of yielding a 50% net return and 50% chance of incurring a loss of 40%. This seems to be a worthwhile investment as it offers an (hypothetical) expected return of 5.0% per iteration.

As such, if a hypothetical player were to repeat such a game a sufficient number of times while staking all her capital (see Figure 1), her hypothetical expected wealth (as displayed by the EV black line) would grow toward infinity; however, she would also almost surely get ruined, even while vastly outperforming other players; (the fate of the 1% top players is illustrated by the golden line: it eventually ends badly).

Figure 1: Simulation results for 100'000 players



Source: TOBAM. For illustration purposes only

The lack of ergodicity of most financial processes

What happens in the above is a tale of increasing wealth concentration: while a select number of alternative futures lead to a flickering of extreme wealth, most paths *and eventually all* lead to ruin. This is also illustrated in Table 1, where we consider the obtained results on a population of N = 100'000 players. The expected value is the probability-weighted average of all possible outcomes whereas the median indicates the wealth of the average player. As the number of tosses increases, the median decreases while the expected value grows, which is a tell-tale of ever-increasing inequality.

Table 1: Wealth distribution metrics on the simulated results for the coin-toss investment game (N=100'000) wealth at start is 10'000.

	After 10 Tosses	After 100 Tosses	After 1000 Tosses
Top 0.1%	$230'660	$47'998'272	$4
Top 1%	$92'264	$3'071'889	$0
Top 10%	$36'906	$12'582	$0
Median (Top 50%)	$5'905	$51	$0
Average (N=100'000)	$16'027	$737'510	$244
Expected Value	$16'289	$1'315'012	$1.54x$10^{25}$

Source: TOBAM. For illustration purposes only

Under such a setting, wealth is accrued multiplicatively, which induces a self-reinforcing loop, where each loss (or gain) worsens (respectively improves) future game conditions, the gist of the matter in the above being that if one loses 40% and then gains 50% (or the reverse), one loses 10% on aggregate (since (1-0.4)*(1+0.5)=0.9), even if the game has a positive Expected Value.

We are here putting the spotlight on the so-called lack of ergodicity of most financial processes. This concept originally stems from the realm of statistical physics, more precisely from the study of gases, when Ludwig Boltzman essentially took the hypothesis that the *long-term time average* of a physical quantity (e.g., the average speed of a given gas particle throughout a long-time horizon) may be obtained by considering its *ensemble average* (i.e., by averaging the speed of all the gas' particles at a given time).

In other words, **a process is deemed ergodic when its ensemble and time averages yield the same results**; and this is often not the case in finance. Mistaking the one for the other would, to paraphrase Nassim Taleb in [3], be tantamount to mistaking the average future of a collection of *one-time* Russian roulette players -for $1million the try- with what happens on average to a player making a collection of such experiments.

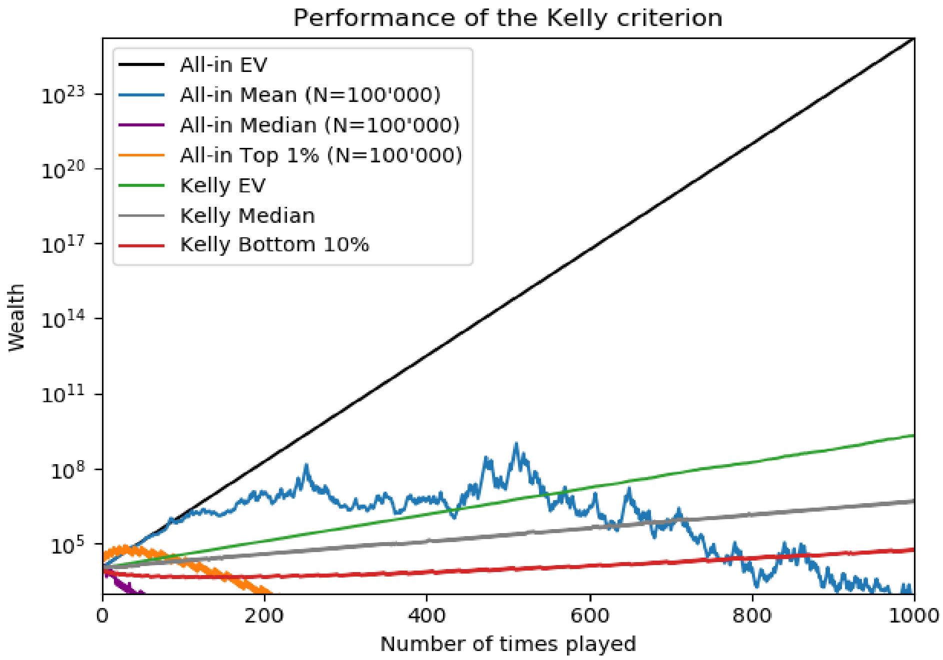
Maximising time average wealth growth rate

The *Kelly criterion*

To avoid falling in the sort of trap illustrated above, a sensible methodology would be for an investor to try and maximise his time average rate of return, i.e., the long-term growth rate of his wealth, instead of focusing on the ensemble-average of his possible futures. This subject is currently being extensively researched by Ole Peters at the LML [4].
A steppingstone in this regard is J. L. Kelly's 1956 paper "A New Interpretation of Information Rate" [5], where he found that under purely multiplicative dynamics, investors should bet a fixed proportion of their wealth at each iteration (the so-called *Kelly criterion*) so as to maximise time average wealth growth rate[2] ; this quantity being asset-specific[3].
The application of the Kelly criterion to our problem comes with the conclusion that 25% of one's capital should be bet on any other coin toss, which yields the following results (Figure 2):

[2] which in *this particular case* is the same problem as maximising the expected logarithm of wealth in the ensemble state
[3] The reader may be interested in knowing that using a lognormal model for securities returns (which is *the* mainstream model) yields an optimal leverage equal to the investment Sharpe ratio divided by its volatility $(\frac{\mu-r}{\sigma^2})$ [6].

Figure 2: Results obtained when applying the Kelly criterion to our investment game (N=100'000)



Source: TOBAM. For illustration purposes only

We see that bottom 10% Kelly criterion investors outperform the top 1% All-in investors as early as T=200, although the all-in strategy has a much higher Expected Value. Eventually all Kelly criterion investors are expected to see their average wealth growth rate converge to about 0.60% per iteration, a far cry from the game's promised 5% EV.

The issue of probability-weighted average

Indeed, the All-in strategy expected value is never consistently attained by any investor because wealth cannot be transferred nor averaged between alternative futures.
The expected value is a probability-weighted average of possible future results, it is not representative of any specific possible future because you cannot access this average. Imagine a planet having at night a temperature of -100°C (-148°F) and +130°C (+266°F) at day: the 'average' temperature is of 15°C (59°F) like in the French Riviera, but this metric is meaningless because should you make a trip to this planet, you would either be completely frozen or burnt, not an average of both [3].

A possible bypass to this problem would be to see a number of investors partaking to uncorrelated versions of the above game create a syndicate, or pool, in order to share their investment results for good or ill. The greater the number of participants, the more each partaker's wealth will approach the EV[4].

Diversification is the solution

[4] See www.farmersfable.org for interactive illustration of the above.

In both cases, the problem of ergodicity is solved by seeking diversification. The lone Kelly investors diversifies his risk-taking through time by deleveraging, which allows him to keep enough capital in case of loss in order to be able to take more risks in the future, the idea being that too much risk taking now impedes your capacity to take risks later on.
The syndicate investing scheme can also be seen as a straightforward application of diversification because each member receives the average result of a set of uncorrelated investments.

The above highlights how crucial proper diversification is for long term investors. An investor wanting to protect his long-term capital growth rate cannot afford to see his investments become overly concentrated on particular risk dimensions, even if this means forsaking short term profits. Doing the contrary would indeed amount to "picking up pennies in front of a steam roller" as the saying goes because it increases the risk of suffering the kind of losses there is no going back from; and all you need is one.

Why low vol isn't low risk

In this sense, long-term risk is a function of how well diversified you are, i.e., to how many risk dimensions you are exposed, rather than of how volatile your portfolio appears to be. To state the obvious, volatility is calculated on an historical basis, a methodology, which is blind to low-frequency high-intensity events. Whereas these very events have a disproportionate impact on the long-term rate of return of overly concentrated portfolios; to the point of transforming a positive EV investment to a Loser's game, as illustrated above.
History is littered with the wrecks of investors having mistaken low volatility for low risk. Such was for instance the fate of Russian government bonds investors at the turn of the century and in general of the whole class of fixed annuities renters, which simply vanished during the first half of the XXth century. The same can be said on the fall of the British land-owning gentry [7], on how the Vanderbilt heirs lost the world's greatest fortune [8], etc…
The canonical demise of LTCM provides a much more recent example of how such considerations may fool the smartest amongst us [9].

As a recommendation, the low hanging fruit consists in equalising the portfolio exposure to the different available independent risk dimensions, which is our aim at TOBAM; by referring to sophisticated concepts such as ergodicity and using tools of higher algebra, we are here only trying to best apply what popular wisdom intuitively understood on the sharing of eggs among many baskets.

To spin the metaphor a little bit further, there may come a time, when an already well diversified investor may actively be searching for hard-to-reach new baskets with which to further boost his diversification. Such a new investment avenue should ideally provide the maximum amount of diversification for the lowest immobilisation of capital. Which means it should be independent, and even insulated, from other traditional financial assets while at the same time being risky enough, when taken on its own.

Bitcoin as a diversifier

A prime example of such an exotic and concentrated source of diversification is Bitcoin, an asset we at TOBAM accordingly developed a keen interest for.
Bitcoin was created in 2008 in the aftermath of the Great Financial Crisis and ensuing Quantitative Easing. As its core, it is a Peer-to-Peer protocol aiming to permit the exchange of value between its participants. The rules are mathematically enforced to prohibit centralisation[5] and to avoid inflation. Compared to other crypto-assets, the Bitcoin protocol is optimised to act as a value storage and settlement system. The ownership and control of bitcoins is determined by a piece of information (essentially a password), which makes the system highly independent and censorship resistant.

[5] I.e. a state in which a single or a few actors may censor or otherwise control the network transactions.

Bitcoins cannot be seized nor frozen without the consent of their owner; as a consequence, it is for instance currently being used by people wanting to flee countries under capital control, like Venezuela, with what little wealth they have left[6]. The use-case of Bitcoin as a hedge is also exemplified by one of its earliest surges, i.e., by the 2013 Cypriot crisis, when bank account holders were subjected to a haircut to alleviate the country's debt burden[7].
In any case, it is an asset which has been engineered to be quintessentially insulated from the traditional financial system. From an empirical perspective, we also notice that it is has not been significantly correlated to traditional financial assets [10].
As such, when carefully weighted it may be used to add one independent risk dimension to any portfolio comprised of traditional financial assets [10].

By adding more effective risk dimensions to its portfolio, be it by investing in additional exotic risk sources or by optimally exploiting the diversification sources available, an investor should improve the long-term growth rate of his capital. To quote Warren Buffet, "in order to succeed, you must first survive"; and proper diversification is a prerequisite of sustained survival.

Black Friday on May 9, 1873 at the Vienna stock exchange, wood engraving from 1873

[6] https://time.com/5486673/bitcoin-venezuela-authoritarian/
[7] https://www.cnbc.com/id/100597242

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See also: Haghani, V., & Dewey, R. (2016). Rational decision-making under uncertainty: Observed betting patterns on a biased coin. Available at SSRN 2856963.

For more information

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The Maximum Diversification® approach, TOBAM's flagship investment process founded in 2006, is supported by original, patented research and a mathematical definition of diversification and provides clients with diversified core exposure, in both the equity and fixed income markets.

In line with its mission statement and commitment to diversification, TOBAM also launched a separate activity on cryptocurrencies in 2017.

TOBAM manages US$10.2 billion (at June 30, 2021). TOBAM's team is composed of 49 professionals.

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